How the Elementary Process Theory corresponds to Special Relativity: 'degrees of evolution' as a curled-up fifth dimension

Marcoen J.T.F. Cabbolet*
Department of Philosophy, Vrije Universiteit Brussel
Pleinlaan 2, 1050 Brussels (Belgium)

Abstract — This self-contained, non-technical paper shows that invariance of the squared interval, arguably the most important theorem of Special Relativity (SR), emerges from the Elementary Process Theory (EPT) if the degrees of evolution, a parameter in the EPT, form an additional spatial dimension that is curled up. The analytical-philosophical reasoning is an integral part of the proof that the EPT corresponds to SR, cf. 'Relativity of Spatiotemporal Charactistics of Inertial Motion in the Universe of the Elementary Process Theory' by M. Cabbolet.

Keywords: Elementary Process Theory; Special Relativity; correspondence principle; fifth dimension; time dilation; antimatter; repulsive gravity

1 Introduction

Recently, the Elementary Process Theory (EPT) has been published as a formal axiomatic system that is potentially applicable as a scheme of universal elementary principles under the condition that the gravitational interaction between matter and antimatter is repulsive [1, 2]. While the EPT makes use of a new formal language for physics and introduces fundamentally new laws of physics, its main issue is that there is insufficient proof that the EPT satisfies the correspondence principle. That is, so far no proof has been presented that one of the theories of modern physics emerges from the EPT—which has led to some critical publications, in essence saying that the EPT is not worthy of further consideration *because* there is no proof of correspondence [3, 4].

The purpose of the present paper is to end this situation and to show that the EPT corresponds with Einstein's special theory of relativity (SR), first published in [5], by showing that the invariance of the squared interval, which is "probably the most important theorem" of SR [6], emerges from the EPT. In SR, the speed of light c is set at its natural value $c = 1$ for all observers, and one considers only inertial (i.e. unaccelerated) observers and their coordinate systems, called inertial reference frames (IRFs). For an observer $\mathcal{O}$ an event $\mathcal{E}$ is an element $\vec{x}$ of a four-dimensional real vector space, and the expression

$$\vec{x} \underset{\mathcal{O}}{\longrightarrow} (x, y, z, t) \tag{1}$$

has to be read as: in the IRF of $\mathcal{O}$, the vector $\vec{x}$ has coordinates (x, y, z, t). We can formulate this theorem then as follows:

Theorem 1.1. *Let, for any displacement of any particle from an event $\mathcal{E}_1$ to an event $\mathcal{E}_2$, the displacement vector $\Delta\vec{x}$ have coordinates $(\Delta x, \Delta y, \Delta z, \Delta t)$ in the IRF of an observer $\mathcal{O}$ and coordinates $(\Delta x', \Delta y', \Delta z', \Delta t')$ in the IRF of an observer $\mathcal{O}'$:*

$$\Delta\vec{x} \underset{\mathcal{O}}{\longrightarrow} (\Delta x, \Delta y, \Delta z, \Delta t) \tag{2}$$

*E-mail: Marcoen.Cabbolet@vub.ac.be

$$\Delta\vec{x} \underset{\mathcal{O}'}{\longrightarrow} (\Delta x', \Delta y', \Delta z', \Delta t') \tag{3}$$

Then, for any two inertial observers $\mathcal{O}$ and $\mathcal{O}'$ the Minkowskian measure η of the displacement vector, denoted by the symbol Δs^2 (the squared interval), is identical:

$$\Delta s^2 = \eta(\Delta\vec{x}, \Delta\vec{x}) = \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2 = \Delta t'^2 - \Delta x'^2 - \Delta y'^2 - \Delta z'^2 \tag{4}$$

□

The precise purpose of this paper is thus to show that this invariance of the squared interval emerges from the EPT, with $\Delta s^2 = 0$ for zero rest mass particles (e.g. photons) and $\Delta s^2 > 0$ for nonzero rest mass particles (e.g. electrons, protons) as in SR. This is not a matter of formal deduction: invariance of the squared interval follows from an analysis of the generic process described by the EPT. The next section presents this analysis; the exposition is self-contained. Sect. 3 presents the resulting relations between spatiotemporal characteristics of the individual processes in the universe of the EPT, and Sect. 4 shows that this corresponds to SR: with that, this paper does the ground laying work for further developments towards a full proof that the EPT satisfies the correspondence principle.

Just like SR only applies under certain conditions, such as absence of gravitational interactions, here too some idealizations have to be made (with '3D' meaning: three-dimensional):

Presupposition 1.2. *For any observer, the following conditions are assumed to be satisfied:*

(i) the spatial dimensions form a 3D Euclidean space at constant value of the non-spatial coordinate(s);

(ii) the distance between two fixed points in 3D space is independent of the non-spatial coordinate(s);

(iii) any displacement of any particle in 3D space and time is linear. □

Arguably, Ps. 1.2 is equivalent to the presupposition that all interactions are negligible. Of course this condition isn't satisfied in practice—which, by the way, is also true for the presuppositions of SR—but still the picture that emerges gives a decent impression of the worldview of the EPT. In the remainder of the text, the term 'squared interval' is used for the real number Δs^2 from SR, and the term 'invariant interval' for the real number $\Delta s = \sqrt{\Delta s^2}$.

2 Analysis

2.1 Background

The EPT consists of seven well-formed formulas (wffs), which are non-logical axioms of a formal axiomatic system, plus a physical interpretation of the individual constants of the axiomatic system as ultimate constituents of a universe. This yields the picture that these seven wffs describe what happens in a *generic individual process* that takes place at supersmall scale in the universe of the EPT; the following predicates then apply:

(i) These seven wffs are *a priori* propositions, in the sense that they are true *before* the process has taken place. This corresponds with the degree of abstractness of the EPT: individual constants referring to ultimate constituents have as value 'a set' without that set being specified any further. That way, individual constants are *designators* of things in the physical universe, without quantitatively representing the state of the designated thing. With this application of the concept of a designator, the a priori propositions are true *regardless of the state of the things referred to.*

(ii) These seven wffs are *synthetic* propositions, in the sense that the relations expressed by these propositions do follow from the essence of the things referred to by the these propositions. For example, one of the seven wffs dictates that every extended particlelike constituent of the universe of the EPT spontaneously transforms into a nonlocal wavelike constituent: it doesn't follow from the essence of these constituents that the one transforms into the other.

(iii) These seven wffs are about what Kant called the *noumenal* universe: these are, thus, about the universe as it is in itself, apart from how it is being observed.[1]

The description of the generic individual process by the EPT thus consists of synthetic a priori propositions on the noumenal universe: if we take this description to be *fundamental*, then all individual processes are essentially the same regardless of the type of interaction that takes place—the EPT corresponds with the idea that there is only one cosmic interaction, of which gravity and electromagnetism are aspects.

The bigger picture is then that in the universe of the EPT, the observable process of evolution can be indexed by a parameter called *degrees of evolution*: there are a finite number of integer-valued degrees of evolution, and at every such degree of evolution there are a finite number of individual processes from that degree of evolution to the next. The generic process described by the EPT is the k^{th} process of the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution.

A main feature of the EPT is that *massive entities* (electrons, positrons, etc.) exhibit stepwise motion: as they alternate between a particlelike state and a wavelike state, they *move* in a wavelike state from one *motionless* particlelike state to the next.[2] See Fig. 1 for an illustration. The generic process described by the EPT thus gives the mechanism for *how* massive entities alternate between particlelike and wavelike states: by one individual process, one massive entity makes one step in its stepwise motion.

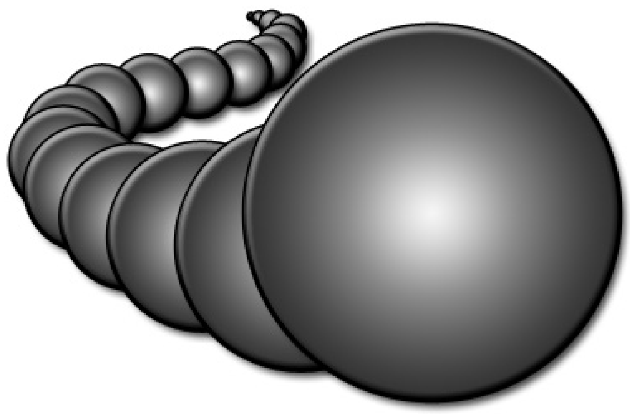


Figure 1: illustration of the stepwise motion of an electron. The balls are abstract representations of successive particlelike states of rest of the electron: in such a state of rest the electron has a definite spatiotemporal position and is devoid of motion. The wavelike states of motion (not depicted) exist 'in between' the states of rest: the electron is then a matter wave spread out over space.

To get from here to the postulates of SR, a number of auxiliary hypotheses are formulated on the basis of an analysis of a process in the universe of the EPT; the following predicates then apply:

(i) These auxiliary hypotheses are *a posteriori* propositions, in the sense that are based on an analysis of the process *after* it has taken place.

(ii) These auxiliary hypotheses are *analytical* propositions, in the sense that they express a meaning that is contained in the synthetic propositions (the seven wffs); however, these analytical propositions cannot be *formally deduced* from the EPT.

(iii) These auxiliary hypotheses are about what Kant called the *phenomenal* universe: these are, thus, about the universe as it is observed.

These analytical a posteriori propositions thus express spatiotemporal characteristics of individual processes in the universe of the EPT in the mathematical language of real analysis: invariance of the squared interval emerges from there. See Fig. 2 for a Toulmin scheme illustrating the emergence of SR from the EPT plus the auxiliary hypotheses.

2.2 Analysis of the generic process

Let's analyze the generic process described by the EPT, that is, the k^{th} process of the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution in the universe of the EPT; we imagine that the process has already

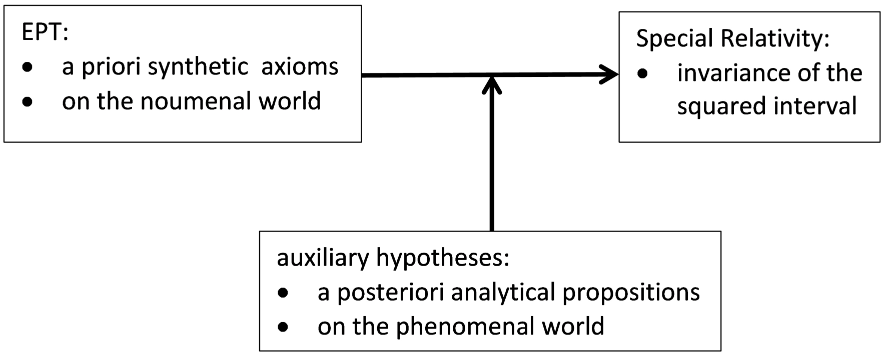


Figure 2: Toulmin scheme illustrating the emergence of SR from the EPT.

happened. We will use the term 'singular event' for events that can be associated with a single position, that is, a single set of spatiotemporal coordinates; the term 'nonsingular event' then refers to an event that is not singular. Recall Ps. 1.2: in this process no nuclear fission, nuclear fusion, or nuclear decay takes place. In terms of events, this one individual process, by which a massive entity (e.g. an electron) makes a single step in its stepwise motion, then went as follows, see Fig. 3 for an illustration:[3]

(1) first, the singular event $\mathcal{E}^0_{n,k}$ took place: by a discrete transition a **point particle**, which at the n^{th} degree of evolution occupied a single position $X_n = (x_n, y_n, z_n)$ in 3D space at a point in time t_n, has transformed into an **extended particle**, which at the n^{th} degree of evolution for a co-moving observer at the point in time t_n occupied a closed ball $\bar{B}(r, X_n)$ in 3D space with radius r and center $X_n = (x_n, y_n, z_n)$. The extended particle is the particlelike state of the massive entity, which in the terminology of the EPT is thus the *extended particlelike phase quantum of the* k^{th} *process of the* n^{th} *to the* $(n+1)^{\text{th}}$ *degree of evolution*, denoted by the symbol ${}^{EP}\Phi^n_k$.[4,5]

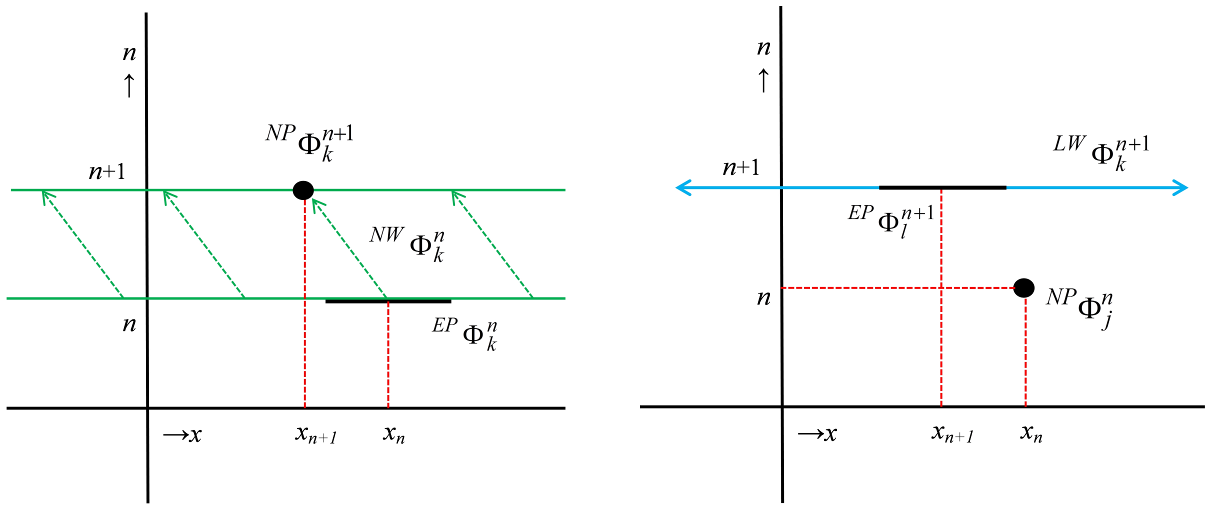
(2) second, the nonsingular event $\mathcal{E}^{(0,1)}_{n,k}$ took place: by a discrete transition, the extended particle has transformed into the wavelike state of the massive entity, which in the terminology of the EPT is the *nonlocal wavelike phase quantum of the* k^{th} *process of the* n^{th} *to the* $(n+1)^{\text{th}}$ *degree of evolution*, denoted by the symbol ${}^{NW}\Phi^n_k$. Such an object is spread out over space *and* time: at every degree of evolution in between the n^{th} and the $(n+1)^{\text{th}}$ degree of evolution, this wavelike state occupied for a co-moving observer the entire 3D space $\mathbb{R}^3$ at one point in time. At every such point in time, the massive particle was a **nonlocal matter wave**.

(3) third, the singular event $\mathcal{E}^1_{n,k}$ took place: at the $(n+1)^{\text{th}}$ degree of evolution the wavelike state has collapsed into a point-particle, which at the $(n+1)^{\text{th}}$ degree of evolution occupied a single position $X_{n+1} = (x_{n+1}, y_{n+1}, z_{n+1})$ in 3D space at a point in time t_{n+1}. In the terminology of the EPT this is the *nonextended particlelike phase quantum of the* k^{th} *process of the* n^{th} *to the* $(n+1)^{\text{th}}$ *degree of evolution*, denoted by the symbol ${}^{NP}\Phi^{n+1}_k$.

(4) fourth, the singular event $\mathcal{E}^{1+}_{n,k}$ took place: at the $(n+1)^{\text{th}}$ degree of evolution the point-particle has emitted a **massless energy quantum**, which in the terminology of the EPT is the *local wavelike phase quantum of the* k^{th} *process of the* n^{th} *to the* $(n+1)^{\text{th}}$ *degree of evolution*, denoted by the symbol ${}^{LW}\Phi^{n+1}_k$.

(5) due to the emission of the massless energy quantum, the point-particle has transformed by a discrete transition into the next particlelike state of the massive entity: this is the singular event $\mathcal{E}^0_{n+1,l}$ for some integer l, which marks the beginning of the l^{th} process of the $(n+1)^{\text{th}}$ to the $(n+2)^{\text{th}}$ degree of evolution. In the terminology of the EPT, by this event $\mathcal{E}^0_{n+1,l}$ the *extended particlelike phase quantum of the* l^{th} *process of the* $(n+1)^{\text{th}}$ *to the* $(n+2)^{\text{th}}$ *degree of evolution*, denoted by the symbol ${}^{EP}\Phi^{n+1}_l$, has arisen.

Analyzing, the latter particlelike state of the massive entity, denoted by ${}^{EP}\Phi^{n+1}_l$, arises **at the same position** X_{n+1} in 3D space were the point-particle denoted by ${}^{NP}\Phi^{n+1}_k$ was; likewise, the earlier



(a) Stages (1)-(3)   (b) Stages (4)-(5)

Figure 3: Two xn-diagrams, with the spatial coordinate x on the horizontal axis and the degrees of evolution n on the vertical axis, depicting the individual process by which an massive entity makes one step in its stepwise motion. In figure (a), showing stages (1) to (3), the black line segment centered at $x = x_n$ depicts the particlelike state of the massive entity at the n^{th} degree of evolution, $^{EP}\Phi^n_k$. The lower horizontal green line depicts the wavelike state of the massive entity that arose from the particle like state by a discrete transition, the green arrows indicate the evolution of the wavelike state in the interval $(n, n+1)$, and the upper horizontal green line depicts the wavelike state of the massive entity just before its collapse; combined, this depicts the nonlocal wavelike phase quantum $^{NW}\Phi^n_k$, which is an object spread out over space and over the interval $(n, n+1)$. The black dot at $x = x_{n+1}$ depicts the point-particlelike state $^{NP}\Phi^{n+1}_k$, which arose by the collapse of $^{NW}\Phi^n_k$. In figure (b), the blue arrow depicts the local wave $^{LW}\Phi^{n+1}_k$ emitted by the point-particlelike state at coordinates $(x_{n+1}, n+1)$ in stage (4) of the process. The black line segment centered at $x = x_{n+1}$ depicts the new particlelike state of the massive entity at the $(n+1)^{\text{th}}$ degree of evolution arising in stage (5) of the process. The black dot at coordinates (x_n, n) depicts the point-particlelike state $^{NP}\Phi^n_j$ that *preceded* the particlelike state of the massive entity at the n^{th} degree of evolution. The two black line segments thus correspond to two consecutive "balls" in Fig. 1.

particlelike state of the massive entity, denoted by $^{EP}\Phi^n_k$, was preceded by such a point-particle at a position X_n in 3D space. So although the two consecutive particlelike states of the massive entity are extended and occupy a region in 3D space, the fact that the preceding point-particles occupy **a single point** in 3D space justifies us to nevertheless state that this step in the stepwise motion of the massive entity involves a 'leap' $X_n \to X_{n+1}$ from **one** position in 3D space at the n^{th} degree of evolution to **one** position in 3D space at the $(n+1)^{\text{th}}$ degree of evolution. Furthermore, due to the finite lifetime of the intermediate wavelike state of motion of the massive entity, denoted by $^{NW}\Phi^n_k$, this step has a *duration*. Summarizing, the following corollary follows from the axioms of the EPT:

Corollary 2.1. *For any observer $\mathcal{O}$, every step in the stepwise motion of a massive entity*

(i) is a 'leap' from a position (x_n, y_n, z_n) in 3D space at the n^{th} degree of evolution to a position $(x_{n+1}, y_{n+1}, z_{n+1})$ in 3D space at the $(n+1)^{\text{th}}$ degree of evolution, where n is an integer;

(ii) has a duration Δt in time. □

In this section we have thus established that an individual process in the universe of the EPT involves a spatiotemporal displacement of a massive entity that is relative to the observer, and a (unit) displacement in degrees of evolution that is observer-independent.

3 Relativity in the universe of the EPT

Consider that a leap of a massive entity for an observer $\mathcal{O}$ has a duration Δt and is a displacement $(\Delta x, \Delta y, \Delta z)$ in 3D space and Δn in degrees of evolution, and for an observer $\mathcal{O}'$ has a duration $\Delta t'$ and is a displacement $(\Delta x', \Delta y', \Delta z')$ in 3D space and $\Delta n'$ in degrees of evolution. Then $\mathcal{O}$ and

$\mathcal{O}'$ may not agree on the spatiotemporal displacement accomplished by this leap, but on account of clause (i) of Cor. 2.1 both observers will agree that this leap involves a unit displacement in degrees of evolution: $\Delta n' = \Delta n = 1$. This aspect is thus observer-independent.

Modeling the supersmall scale in the universe of the EPT with the Planck scale, the basic idea is then that this unit displacement in degrees of evolution is a displacement of a Planck length **in an <u>additional</u> spatial dimension**. So we consider Planck units, that is, we consider that Planck length ($\approx 1.6 \cdot 10^{-35}$ m) and Planck time ($\approx 5.4 \cdot 10^{-44}$ s) are scaled to 1. An observer who lives in the universe of the EPT thus lives in a **five-dimensional spacetime**, in which the 'degrees of evolution' form an additional dimension—note that this **by no means** implies that inhabitants of the universe of the EPT can *freely move* in five dimensions. Furthermore, a philosophy of time is then that the duration of a leap is nothing but the Euclidean measure of the spatial displacement. We summarize this in the following proposition:

Proposition 3.1. *For any observer $\mathcal{O}$ and for every single leap of every massive entity with duration Δt and spatial displacement $(\Delta x, \Delta y, \Delta z)$ in Planck units, we always have*

$$\Delta n = 1 \tag{5}$$

$$\Delta t^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 + \Delta n^2 \tag{6}$$

for the displacement in degrees of evolution Δn. □

The next point is then that the dimension of 'degrees of evolution' has to be **curled up**. This can be illustrated with the simplest of examples—recall clause (iii) of Ps. 1.2: the displacement in consecutive leaps of a nonzero rest mass entity remains the same.

Example 3.2. Consider an observer $\mathcal{O}$, for whom a massive entity $\mathcal{P}$ makes the following leaps in xnt-space (suppressing spatial coordinates y and z):

- $(2,1,1) \rightarrow (2\frac{3}{4}, 2, 2\frac{1}{4})$, $(2\frac{3}{4}, 2, 2\frac{1}{4}) \rightarrow (3\frac{1}{2}, 3, 3\frac{1}{2})$, $(3\frac{1}{2}, 3, 3\frac{1}{2}) \rightarrow (4\frac{1}{4}, 4, 4\frac{3}{4})$, $(4\frac{1}{4}, 4, 4\frac{3}{4}) \rightarrow (5,5,6)$

For the observer $\mathcal{O}$, each of the leaps is thus a displacement $(\Delta x, \Delta n) = (\frac{3}{4}, 1)$ in the xn-plane, and has thus a duration Δt of $\sqrt{\Delta x^2 + \Delta n^2} = \frac{5}{4}$. See Fig. 4 for an illustration. For the observer $\mathcal{O}$,

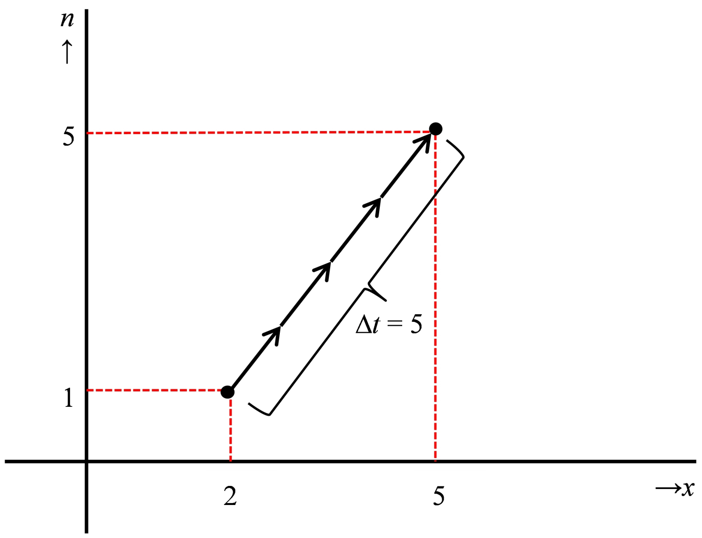


Figure 4: The four leaps of the particle $\mathcal{P}$ in the world of $\mathcal{O}$, depicted in a xn-diagram. Horizontally the spatial coordinates x, vertically the degrees of evolution n. The arrows represent the leaps.

the entity $\mathcal{P}$ has thus a speed $v = \frac{\Delta x}{\Delta t} = \frac{3}{5} = 0.6$ in the positive x-direction. Now consider that a second massive entity $\mathcal{Q}$ was at $t = 0$ at the position $(x, n, t) = (5, 0, 0)$, and is at rest at $x = 5$. Ignoring particle sizes, we **know** that these entities will collide $t = 6$—let there be no doubt about that. But at $t = 6$, entity $\mathcal{P}$ is at the position $(x, n, t) = (5, 5, 6)$ while entity $\mathcal{Q}$ is at the position $(x, n, t) = (5, 6, 6)$. But that means that a collision is only possible if $(5, 5, 6)$ and $5, 6, 6)$ are *physically* the same point. **So if the dimension of degrees of evolution wasn't curled-up, the particles $\mathcal{P}$ and $\mathcal{Q}$ would miss each other!** □

That brings us to the following proposition, where $x \equiv y \pmod 1$ means $|y - x| \in \mathbb{N}$, that is, x is congruent to y modulo 1:

Proposition 3.3. *The degrees of evolution form a curled-up dimension. That is, the dimension of degrees of evolution can be modeled by the set $\mathbb{R}$ in Planck units together with an equivalence relation $\sim$ given by*

$$x \sim y \Leftrightarrow x \equiv y \pmod 1 \tag{7}$$

where $x \sim y$ is to be interpreted as 'x and y are ***physically*** *the same point'.* □

The equivalence relation $\sim$ transforms the dimension of degrees of evolution into a "topological circle" whose "circumference" is one.[6] The idea of a curled up dimension is not new, and has first been proposed by Klein [8].

Remark 3.4. Because of this feature of being curled-up, the view that the degrees of evolution form a fifth dimension differs **fundamentally** from Bordé's view that the fifth dimension is formed by the proper time coordinates [9]. This can be seen as follows. Suppose that degrees of evolution and proper time are physically the same thing: then an essential feature of the proper time dimension would be that it is curled up. But then the dimension time would also be curled up: relativistic effects can cause time dilation, but cannot change such an essential feature of the dimension. But then we would constantly be colliding with our past selves: a point later in time would then physically be the same as a point earlier in time—cf. Prop. 3.3. But this is not what happens in the real world, so the dimension of degrees of evolution cannot be identical to a proper time dimension. □

We haven't said a word yet about massless particles. But recall that our mission here is to prove correspondence of the EPT to SR: our mission is **not** to build an interaction theory. So we conveniently ignore that massless energy quanta may have wavelike properties, and we introduce them in the following proposition as dimensionless particles:

Proposition 3.5. *Suppose all massless energy quanta, denoted by symbols ${}^{LW}\Phi_k^{n+1}$ in the EPT, can be modeled by dimensionless (i.e. size-less) particles. Then, for any observer $\mathcal{O}$ and for any displacement $(\Delta x, \Delta y, \Delta z, \Delta n, \Delta t)$ of any massless energy quantum in Planck units, we always have*

$$\Delta n = 0 \tag{8}$$

$$\Delta t^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 + \Delta n^2 \tag{9}$$

where Δn is the displacement in degrees of evolution. □

These three analytical propositions 3.1, 3.3, and 3.5, which express the basic relations between space and time in the universe of the EPT, are the auxiliary hypotheses meant in Sect. 2.1. In the framework of the EPT, each of these expresses in concrete mathematical language a physical meaning of the (abstract) axioms of the EPT.

4 Discussion

4.1 Correspondence to standard SR

Consider, in standard SR, an inertial observer $\mathcal{O}$ and a particle co-moving with the observer. Consider two events $\mathcal{E}_1$ and $\mathcal{E}_2$ on the world line of the particle, such that we have for the displacement vector $\Delta\vec{x}$ in Planck units

$$\Delta\vec{x} \underset{\mathcal{O}}{\longrightarrow} (0, 0, 0, 1) \tag{10}$$

For the invariant interval we then get

$$\Delta s = \sqrt{\eta(\vec{x}, \vec{x})} = 1 \tag{11}$$

But that means that for the displacement of the particle from $\mathcal{E}_1$ to $\mathcal{E}_2$, we have the following for the associated displacement vector $\Delta\vec{x}' = (\Delta x', \Delta y', \Delta z', \Delta t')$ in the IRF of an arbitrary observer O'

$$\Delta t'^2 = \Delta x'^2 + \Delta y'^2 + \Delta z'^2 + 1 \tag{12}$$

But this is precisely what we get when we substitute Eq. (5) in Eq. (6). We summarize this result in the following lemma:

Lemma 4.1. *For any observer and for any leap of any nonzero rest mass entity in the universe of the EPT in Planck units, the displacement in degrees of evolution is exactly identical to the invariant interval of the associated spatiotemporal displacement vector in Minkowski space.* □

Remark 4.2. It is emphasized that we do **not** state that the change in degrees of evolution is a *physical interpretation of the invariant interval*: what has been stated in Rem. 3.4 also goes here. That is, in SR the invariant interval **lacks** the property $\Delta s_1 \sim \Delta s_2 \Leftrightarrow |\Delta s_1 - \Delta s_2| \in \mathbb{N}$, which **is** a property in the dimension of degrees of evolution. So we refrain from adding a property to the invariant interval that is nonexisting in SR: we merely say that at this fundamental level the change in degrees of evolution, which occurs in the framework of the EPT, is numerically identical to the invariant interval of SR. □

The same reasoning can be applied to massless particles. In standard SR, any displacement vector $\Delta\vec{x}$ of any massless particle in the IRF of any inertial observer $\mathcal{O}$ satisfies

$$\Delta s = \sqrt{\eta(\vec{x}, \vec{x})} = 0 \tag{13}$$

so that we get

$$\Delta t'^2 = \Delta x'^2 + \Delta y'^2 + \Delta z'^2 + 0 \tag{14}$$

But this is precisely what we get when we substitute Eq. (8) in Eq. (9). We summarize this result in the following lemma:

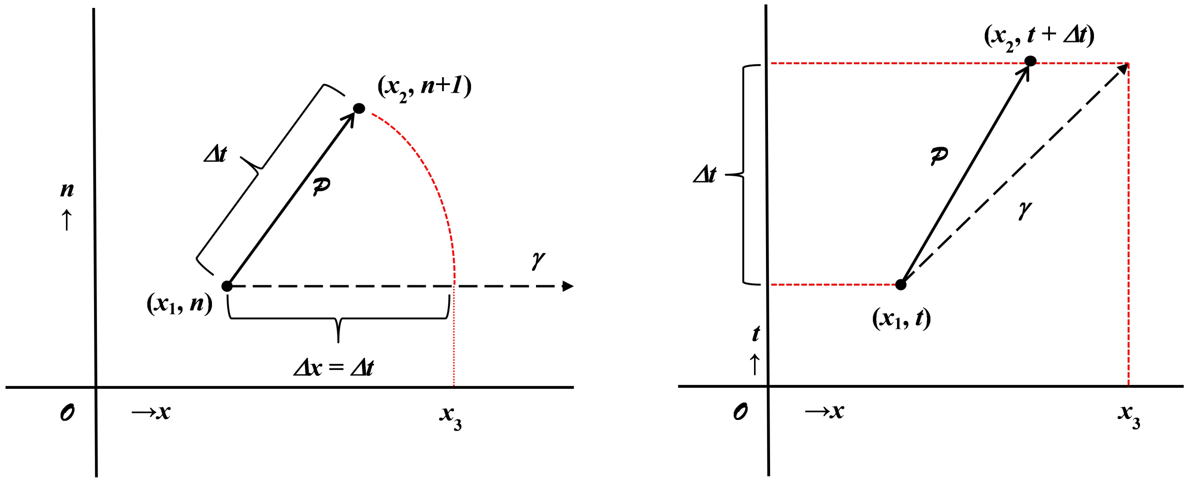
Lemma 4.3. *For any observer and for any displacement of any massless particle in the universe of the EPT in Planck units, the displacement in degrees of evolution is exactly identical to the invariant interval of the associated spatiotemporal displacement vector in Minkowski space.* □

With Lemmas 4.1 and 4.3, we have reproduced the desired theorem of SR, Th. 1.1, from the EPT. That is, we have shown that in the universe of the EPT, the Minkowskian measure Δs^2 of a displacement $(\Delta x, \Delta y, \Delta z, \Delta t)$ of a particle in 3D space and time is observer-independent, with $\Delta s^2 > 0$ for nonzero rest mass particles and $\Delta s^2 = 0$ for zero rest mass particles like in SR:

this is the correspondence of the EPT with SR.

To conclude this section the philosophy of time, formalized by the Eqs. (6) and (9), is illustrated with an example demonstrating that no clock can be co-moving with a massless particle (e.g. a photon) in the universe of the EPT. Since any clock is made up of nonzero rest mass entities, the clock needs **more time** than the zero rest mass particle for the same displacement in space: $\Delta n = 1$ in Eq. (6), and $\Delta n = 0$ in Eq. (9).

Example 4.4. Consider for an observer $\mathcal{O}$, a nonzero rest mass entity $\mathcal{P}$ emits a massless particle γ in positive x-direction from the position with coordinates (x_1, n) in the xn-plane, and makes a leap to the position $(x_2, n+1)$ with $x_2 > x_1$. The duration of the leap of $\mathcal{P}$ is $\Delta t = \sqrt{\Delta x^2 + \Delta n^2} = \sqrt{\Delta x^2 + 1}$. The trajectory of the massless particle γ in the xn-plane is the half line $(x_1 + \lambda, n)$ with $\lambda > 0$, and with a pair of compasses we can easily determine the x-coordinate x_3 where γ is located after a time Δt has passed—the duration of the leap of $\mathcal{P}$. It is always the case that $x_3 > x_2$, so in the xt-plane a massless particle always runs away from a nonzero rest mass entity. See Fig. 5 for an illustration. □



(a) xn-plane of $\mathcal{O}$ (b) xt-plane of $\mathcal{O}$

Figure 5: Illustration of a massless particle γ moving away from a nonzero rest mass entity $\mathcal{P}$. In figure (a), the dashed arrow to the right is the trajectory of γ in the xn-plane. The black arrow between the two dots in the xn-plane of observer $\mathcal{O}$ depicts the leap $(x_1, n) \to (x_2, n+1)$ of $\mathcal{P}$. The duration Δt is indicated, and the dashed red curve shows how the position (x_3, n) of γ after a time Δt can be determined with a pair of compasses. The xt-diagram (b) shows that γ moves away from $\mathcal{P}$.

4.2 Main implication

The main implication of the present result is that it shows that the EPT is consistent with the Michelson-Morley experiment [10] and with observations of time dilation. The following example elaborates on the consistency of the EPT with observations of prolonged lifetimes of fast muons.

Example 4.5. From observations it is known that "slow' muons produced in a laboratory have a lifetime of about $2.2 \cdot 10^{-6}$ s, but that "fast" muons have been observed to live longer in accordance with SR, cf. [11]. So in the framework of the EPT we can put that a muon decays *in a fixed number of interactions*, in casu meaning that it only exists for some $4 \cdot 10^{37}$ degrees of evolution: the lifetime in seconds then follows from clause (iii) of Post. ??. See Fig. 6 for an illustration: if the trajectory of a muon in the nx-plane of the observer has zero spatial displacement ($\Delta x = 0$) then the duration of the trajectory is $\Delta t = 2.2 \cdot 10^{-6}$ s; if however the trajectory of a muon in the nx-plane has a sufficiently large spatial displacement, then the duration of the trajectory can becomes five times larger, giving $\Delta t = 11 \cdot 10^{-6}$ s. □

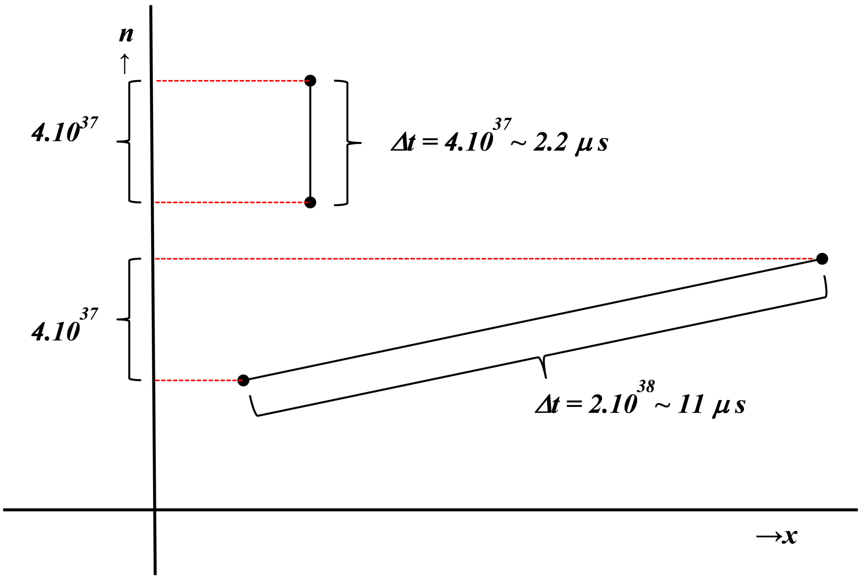


Figure 6: Trajectories of a slow and a fast muon in the xn-plane of an observer $\mathcal{O}$. Horizontally the spatial coordinates x, vertically the degrees of evolution n. The durations of the trajectories are indicated.

4.3 On presuppositions

A presupposition of SR is that 'spatial distance' is measured by *rods*, and 'time' is measured by *clocks*. In fact, in an IRF of an observer $\mathcal{O}$ there are supposed to be synchronized clocks at every point on the x-axis. The space-time diagram in Fig. 7 shows the world lines of five of these clocks.

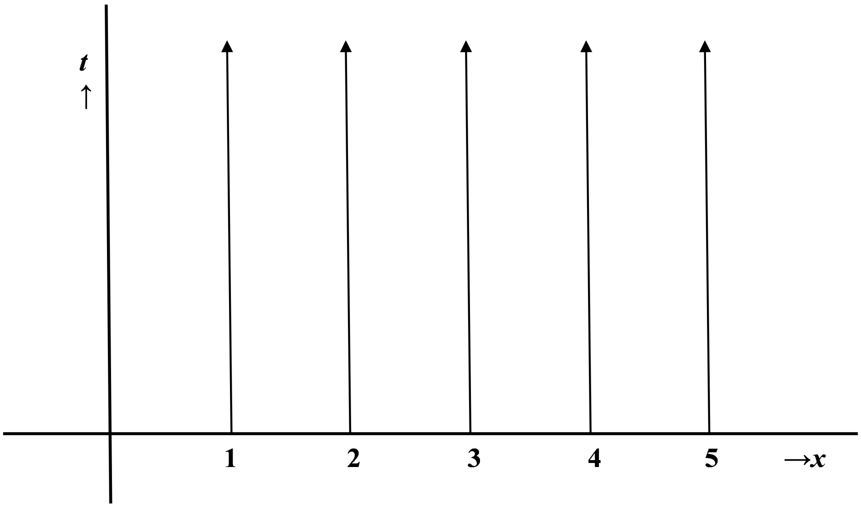


Figure 7: World lines of clocks at $x = 1$, $x = 2$, $x = 3$, $x = 4$, and $x = 5$ in the IRF of an observer $\mathcal{O}$.

Now consider a unit $\Delta x = 1$ to correspond to the Planck distance ($\approx 10^{-35}$ m), and suppose that we use 1 cm to draw this unit on paper in a figure: a real clock that measures, say, 10 cm then corresponds to some 10^{33} m on paper, which means that we would need a piece of paper of more than the size of our galaxy ($\approx 10^{21}$ m) just to draw the spatial extension of the clock—note that this remains true when the clock is made up of a single hydrogen atom (size: 10^{-10} m). So it becomes preposterous to speak of a clock that is co-moving with the observer "at $x = 2$", or to speak of two clocks co-moving at a distance $\Delta x = 1$, if we are talking about the Planck scale: Fig. 7, which shows world lines of real clocks, is thus only meaningful at a macroscopic scale where the size of a clock is negligible compared to the unit $\Delta x = 1$. So we still can apply the idea of a *space-time diagram* to the Planck scale, but we will then have to dismiss the idea that world lines at constant x such as in Fig. 7 can represent trajectories of real clocks moving through space and time.

A second presupposition of SR is that *time* can be measured with infinite accuracy. In the universe of the EPT, however, any time measurement requires at least one interaction, and thus involves *at least* one individual process as described by the EPT (since a clock is itself subject to those processes). That means that there is an **absolute error** in any time measurement of the duration of an individual process (which below will be equated to a Planck time). And that means that if the EPT predicts that there will occur a particle on some position x at, say, $t = 6$ in the IRF of an observer $\mathcal{O}$, then $\mathcal{O}$ **cannot possibly** verify that prediction with infinite accuracy.

Likewise, the presupposition of SR that *position* can be measured with infinite accuracy has to be dismissed at Planck scale: in the end, any device consists of at least one atom of finite size, which means that an infinitely precise position measurement is an illusion. So if we want to talk about occurrences of the point-particlelike states, as described in stage (3) of the individual processes in Sect. 2.1, as events with coordinates in 3D space and time that can be drawn in a space-time diagram, then we will have to dismiss the idea that the time coordinate t and the spatial coordinates (x, y, z) in 3D space of such an event can be *experimentally verified* with infinite accuracy. In other words: there is nothing wrong with associating a precise position to a particlelike state of e.g. an electron, but how, if at all, it can be experimentally verified that the electron is there in that state at that time, and how such a measurement will influence the state of the electron, are then different questions are left aside in the present study.

4.4 Conclusions

The main conclusion is that this paper has shown that the EPT corresponds to SR: with that, the EPT has withstood a theoretical test. This result doesn't follow from a mathematical model of the things that occur in the universe of the EPT, but from fundamental considerations about the dimensionality of the world of an observer and the spatiotemporal characteristics of the elementary processes described by the EPT.

This correspondence renders the EPT in principle consistent with the null result of the Michelson-Morley experiment, which led to the acceptance of SR. That doesn't mean that there is no aether in the universe of the EPT: it only means that this aether does not have the properties of the aether of classical mechanics. It also renders the EPT consistent with the various experimental data confirming the existence of time dilation: this has been discussed in Ex. 4.5.

To keep things simple some conditions have been assumed that are never fulfilled in reality, but that is also true for SR: the bottom line is that the present result is a breakthrough in this research program, aimed at a mathematical model of the universe of the EPT at Planck scale.

Notes

[1]See [7] for the reasoning on how knowledge of the noumenal universe is possible.

[2]To avoid confusion between rest-mass-having *particles* and *particlelike states*, we will speak of *massive entities* and *particlelike states* instead.

[3]For a full description of the process in the formalism of the EPT, the reader is referred to the Annalen papers [1, 2] or the dissertation [7].

[4]In the symbol ${}^{EP}\Phi^n_k$ the Greek letter Φ stands for 'phase quantum' (a primitive notion), the left superscript indicates the type of phase quantum (e.g. *EP*: **e**xtended **p**articlelike), the right superscript n is an integer indicating the integer-valued degree of evolution at which it is created, and the right subscript k is the number of the individual process in which the phase quantum participates.

[5]About the term 'extended particlelike phase quantum', the following. A 'particle' is an object whose size is negligible compared to its motion. Here at supersmall scale, however, we are talking about very, very small leaps. Consider a proton leaping a Planck length: given the proton's charge radius, this is (approximately) proportional to considering that a thing the size of our galaxy moves 1 cm. So the size of the thing is then no longer negligible compared to its motion: the term 'particle' then no longer applies. Therefore, this state is given the name 'extended particlelike phase quantum'. So the term 'extended particle' in the analysis is a slight abuse of language.

[6]One can think of the interval $[0, 1\rangle$ forming a ring with circumference 1, and the real numbers being wound on it like a wire is wound on a bobbin: then one gets the relation $n_1 \equiv n_2 \Leftrightarrow |n_1 - n_2| \in \mathbb{N}$ meaning n_1 and n_2 are physically the same point.

References

[1] M.J.T.F. Cabbolet, *Ann. Phys. (Berlin)* **522**, 699-738 (2010)

[2] M.J.T.F. Cabbolet, *Ann. Phys. (Berlin)* **523**, 990-994 (2011)

[3] W. van Joolingen, "Voor doctor Cabbolet valt het doek" (in Dutch; English translation: The curtain falls for doctor Cabbolet), *Volkskrant, Edition October 28*, 30 (2011)

[4] J.W. Nienhuys, "Een bizarre dissertatie" (in Dutch; English translation: A bizar dissertation), *Skepter* **27**(1), 10-11 (2015)

[5] A. Einstein, *Ann. Phys. (Leipzig)* **17**, 891-921 (1905)

[6] B.F. Schutz, *A first course in general relativity*, Cambridge University Press, Cambridge (1990)

[7] M.J.T.F. Cabbolet, *Elementary Process Theory: axiomatic introduction and applications*, dissertation, Free University of Brussels (2011)

[8] O. Klein, *ZS f. Physik* **37**, 895-906 (1926)

[9] C.J. Bordé, *Phil. Trans. Roy. Soc. A* **363**, 2177-2201 (2005)

[10] A.A. Michelson, E.W. Morley, *Am. J. Sci.* **34**, 333-345 (1887)

[11] B. Rossi, D.B. Hall, *Phys. Rev.* **59**(3), 223-228 (1941)