A 5D categorical model of the Elementary Process Theory incorporating Special Relativity

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Abstract — One of the two main issues with the Elementary Process Theory (EPT), a first-order scheme with a speculative interpretation as elementary principles, is that there is thus far no proof that it agrees with existing knowledge. The objective of this paper is to show that the EPT agrees with the knowledge obtained from the successful predictions of Special Relativity (SR). For that matter, the notion of a model of the EPT is identified with a category, whose objects are set-theoretic models of the EPT, and whose arrows are isomorphisms between these models: each object is then associated with a model of the universe of the EPT in the reference frame of an observer, and each arrow with a transformation between coordinate systems of different observers. A 5D categorical model of the EPT that incorporates SR is then fully specified: objects in the universe of the EPT are modeled as point-particles, γ-rays, or time-like strings, all represented by integrable hyperreal functions on a pseudo-Riemannian 5-manifold with a compact fifth dimension. Processes of inertial motion, Bremsstrahlung, and laser cooling are described, and fundamental differences with other 5D theories are discussed. In addition, the intended relevance of the EPT for physics as a grand unifying scheme is precisely defined. The main conclusion is that the EPT has been proven to agree with SR by specifying a categorical model of the EPT, which is a new application of category theory to physics.

1 Introduction

The Elementary Process Theory (EPT) is a first-order scheme together with a (speculative) physical interpretation given in the form of interpretation rules, which yields the view that the axioms of the EPT are non-classical, non-quantum, elementary principles in a (hypothetical) universe with the feature that massive antiparticles are *repulsed* by the gravitational field of bodies of ordinary matter [1, 2, 3]. The question is then: is this relevant for physics? There are then two main issues with the EPT, both mentioned in [1], which are causes for a genuine concern that the answer to that question is 'no':

(i) the EPT has in essence been developed from a Gedankenexperiment with an outcome (matter-antimatter repulsive gravity) that cannot possibly be true from the perspective of modern physics;

(ii) thus far there is no proof that the EPT agrees with existing knowledge of the physical world.

Concerning the first issue, the crux is that the theoretical arguments against repulsive gravity—see [4, 5] for an overview—lean on the assumption that theories of modern physics are valid beyond their established area of application. But as Feynman already remarked, "experiment is *the sole judge* of scientific truth" [6]. The issue whether or not repulsive gravity exists will thus have to be decided by experimental research; the current state of affairs is then that there are at least four sizeable experimental projects going on to establish the coupling of massive antimatter particles with the gravitational field of the earth: three projects at CERN using anti-hydrogen, AEgIS [7], GBAR [8], and ALPHA [9], and one at the PSI using muonium [10].

Concerning the second issue, a difficulty is that the mathematical framework of the EPT makes no contact with the mathematical framework of existing theories: it is therefore impossible to show that the EPT agrees with existing knowledge by proving what Rosaler called a *formal reduction* [11], that is, by proving that the EPT reduces to an existing theory by applying some limiting procedure. But even though formal reduction is excluded, it nevertheless remains to be proven that the EPT *somehow* agrees with existing knowledge: this is a theoretical test for the EPT. That being said, the purpose of this paper is to show that the EPT agrees with Special Relativity (SR), first published in [12].

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The outline of this paper is as follows. The next section elaborates on the method by which it is shown that the mathematically abstract EPT agrees with the mathematically concrete SR. In particular, this section introduces the new notion of a *categorical model* of the EPT: the idea is that the sentence ‘the EPT agrees with SR’ means ‘the EPT has a categorical model that incorporates SR’—a negative result of this study would thus have been that no such categorical model exists. Sect. 3 thereafter introduces the main result of this study: a five-dimensional (5D) categorical model of the EPT. Sect. 4.1 shows that this corresponds to standard SR, Sect. 4.2 discusses its relation to other 5D theories, Sect. 4.3 formalizes processes of inertial motion, Bremsstrahlung and laser cooling in the present framework, and Sect. 4.4 clarifies the intended relevance of the EPT for physics. Finally, Sect. 4.5 states the conclusions. The remainder of this introduction lists some basic mathematical notations and definitions that are used throughout this paper.

Notation 1.1 We will use the symbols $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$ respectively for the sets of natural numbers, integers, rational numbers, real numbers, and complex numbers, and $\mathbb{U}$ for the unit interval $[0, 1) \subset \mathbb{R}$. The floor function $\lfloor . \rfloor : \mathbb{R} \to \mathbb{Z}$ is given by $\lfloor x \rfloor = \max\{y \in \mathbb{Z} \mid y \leq x\}$. For any $x \in \mathbb{R}$ and $y \in \mathbb{U}$, the expression $x \equiv y \pmod 1$ means: x is congruent to y modulo 1, that is, $|x - y| \in \mathbb{N}$. □

Definition 1.2 The space $(\mathbb{U}, +_1, \cdot)$ consists of the set $\mathbb{U}$ together with the operations addition $+_1 : \mathbb{U} \times \mathbb{U} \to \mathbb{U}$ and scalar multiplication $\cdot : \mathbb{R} \times \mathbb{U} \to \mathbb{U}$, which are defined as follows for any $a, b, c \in \mathbb{U}$ and $x \in \mathbb{R}$:

$$a +_1 b = c \Leftrightarrow (a + b) \equiv c \pmod 1 \tag{1}$$

$$x \cdot a = b \Leftrightarrow xa \equiv b \pmod 1 \tag{2}$$

where the terms ‘$a + b$’ and ‘xa’ in Eqs. (1) and (2) refer to $a + b, xa \in \mathbb{R}$. □

The binary structure $(\mathbb{U}, +_1)$ is isomorphic to the abelian group $(S^1, \cdot)$, the set $S^1 = \{z \in \mathbb{C} \mid |z| = 1\}$ of complex numbers of norm 1 under multiplication. $(\mathbb{U}, +_1)$ is thus an abelian group; for any $a \in \mathbb{U}$, we have thus an element ‘$-a$’ such that $a +_1 (-a) = 0$. Furthermore, $1 \cdot a = a$ for any $a \in \mathbb{U}$.

Corollary 1.3 The space $(\mathbb{U}, +_1, \cdot)$ is not a vector space.

Proof A vector space must satisfy the axiom $\alpha \cdot (a + b) = \alpha \cdot a + \alpha \cdot b$ for any scalar α and any vectors a and b. However, here we have $\frac{3}{2} \cdot (\frac{1}{2} +_1 \frac{1}{2}) = \frac{3}{2} \cdot 0 = 0$ but $\frac{3}{2} \cdot \frac{1}{2} +_1 \frac{3}{2} \cdot \frac{1}{2} = \frac{3}{4} +_1 \frac{3}{4} = \frac{1}{2}$. Q.E.D. □

Definition 1.4 Let $(\mathbb{R}, T)$ be the real line in the standard topology. Let the function $\varrho : \mathbb{R} \to \mathbb{U}$ be given by $\varrho : x \mapsto y \Leftrightarrow x \equiv y \pmod 1$. Then the **standard topology** on $\mathbb{U}$, notation: $T_{\mathbb{U}}$, is the topology coinduced on $\mathbb{U}$ by ϱ. □

Note that ϱ does not yield a *compactification*, because ϱ is not injective. We will use the function ϱ throughout this paper.

Corollary 1.5 For any $x \in \mathbb{U}$, the set $\mathbb{U} \backslash \{x\}$ is open in $\mathbb{U}$.

Proof Consider the open subset $(x - 1, x)$ of $\mathbb{R}$. Then by Def. 1.4, $\varrho[(x - 1, x)]$ is open in $\mathbb{U}$. But $\varrho[(x - 1, x)] = [0, x) \cup (x, 1) = U \backslash \{x\}$ as requested. Q.e.d. □

The space $(\mathbb{U}, T_{\mathbb{U}})$ is homeomorphic to the circle S^1 in its standard topology: $(\mathbb{U}, T_{\mathbb{U}})$ is thus a path connected, compact Hausdorff space.

2 Method

The EPT is a first-order theory[1] from the syntactic point of view, but the crux is that the abstract/concrete distinction from mathematics applies when comparing the EPT and SR. On the one hand, the EPT is a *mathematically abstract* theory that applies the notion of a designator: a constant of the EPT is an abstract set—that is, a set whose elements are not specified—and this constants *designates* an ultimate constituent of the universe *without* representing the state of that ultimate constituent. For example, for integers n and k the constant ${}^{EP}\Phi_k^n$ is an abstract set that refers to the *extended particlelike phase quantum occurring in the* k^{th} *process from the* n^{th} *to the* $(n + 1)^{\text{th}}$ *degree of evolution*: of all individual processes in the universe of the

[1] It is ‘first-order’ in the sense of first-order logic: the adjective ‘first-order’ does **not** refer to first-order differential equations.

EPT, only one is the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution, and in that process only one extended particlelike phase quantum occurs. So the constant ${}^{EP}\Phi_k^n$ refers to a unique object in the universe of the EPT, yet at this degree of abstractness it contains no information about its properties such as position, momentum, etc. Compare the designator 'queen of the Netherlands': it is clear, at least in 2016, to which individual this refers—to wit: Queen Maxima—yet the designator contains no information about position or momentum of that individual. SR, on the other hand, is *mathematically concrete*: an element of Minkowski space is precisely specified and quantitatively represents a position in spacetime that can be measured by clocks and rods—further elaboration is omitted.

To prove that the EPT agrees with SR, one of the tools from formal logic that we have at our disposal is to specify a set-theoretic model of the EPT, that is, an interpretation of the constants and axioms of the EPT in a concrete set-theoretic domain, cf. [13]: the idea is to model a designator like ${}^{EP}\Phi_k^n$ by a concrete mathematical representation of the state of the designated object. However, as soon as we model a mathematically abstract set that *designates* a physical thing by a mathematically concrete set that *represents* the state of that thing (including its properties), then we have to consider the fact that different observers may model the same designator differently. For example, we may want to model the aforementioned designator ${}^{EP}\Phi_k^n$ as a point-particle with a definite position and a definite momentum, but different observers will assign a different position and a different momentum to the same physical object—ergo, a *different observer* then corresponds to a *different set-theoretic model* of the EPT. So if we are strict in applying the syntactical view on theory and models, then specifying a single set-theoretic model of the EPT is clearly insufficient to show agreement with SR. That motivates us to introduce the new notion of a *categorical model of the EPT*, which identifies a model of the EPT with a category:

Definition 2.1 A **categorical model of the EPT** is a category $\mathscr{C}$ such that

(i) the collection of objects of $\mathscr{C}$ is a family $\{M_i\}_{i\in F_1}$ of set-theoretical models of the EPT, so that any model M_p in $\{M_i\}_{i\in F_1}$ is a structure $M_p = \langle |M_p|, I_p(M_E), I_p(R)\rangle$ for the EPT, with

- I_p being the interpretation function from the language of the EPT to the language of M_p;
- $|M_p|$ being the universe of individuals of M_p, which for any constant ϕ of the EPT contains an interpretation $I_p(\phi) \in |M_p|$ that is *mathematically concrete*;
- $I_p(M_E) \subset |M_p|$ being the interpretation of the unary existence relation M_E of the EPT ;
- $I_p(R) \subset |M_p| \times |M_p| \times |M_p|$ being the interpretation of the ternary relation R of the EPT.

(ii) the collection of arrows of $\mathscr{C}$ is a family $\{T_j\}_{j\in F_2}$ of structure isomorphisms, so that for any arrow T_k in $\{T_j\}_{j\in F_2}$ there is a domain $M_p \in \{M_i\}_{i\in F_1}$ and a codomain $M_q \in \{M_i\}_{i\in F_1}$ such that

- T_k bijectively maps the universe of individuals $|M_p|$ to the universe of individuals $|M_q|$;
- $T_k(\alpha) \in I_q(M_E) \Leftrightarrow \alpha \in I_p(M_E)$;
- $(T_k(\alpha_1), T_k(\alpha_2), T_k(\alpha_3)) \in I_q(R) \Leftrightarrow (\alpha_1, \alpha_2, \alpha_3) \in I_p(R)$.

□

This Def. 2.1 provides us with a new tool: the method to prove that the EPT agrees with SR now comes down to (a) specifying a categorical model of the EPT, and (b) showing that this corresponds to SR. This is the method that has been applied in the present study; the main result, presented in the next section, is the categorical model of the EPT $\mathscr{C}_{SR}$—this is the 5D categorical model to which the title of this paper refers.

The specification of the category $\mathscr{C}_{SR}$ is straightforward—see [1, 3] for a list of the constants of the EPT that have to be interpreted—but some elaboration is in place on how the components of the universe of the EPT have been modeled. It has to be taken that the EPT is a mathematically abstract theory that states elementary principles in terms of ultimate components but without reference to any coordinate system of an observer, while each model M_p in $\{M_i\}_{i\in F_1}$ is a mathematically concrete interpretation of these principles in the reference frame of an inertial observer. Recall that the universe described by the EPT consists of *world and antiworld*: a component of this universe is designated by a 2×1 matrix $\begin{bmatrix} \phi \\ \overline{\phi} \end{bmatrix}$, where the abstract set ϕ designates a constituent of the world and the abstract set $\overline{\phi}$ a constituent of the antiworld—observers who live in "our" forward time-direction thus only observe a manifestation (i.e., a state) of the constituent ϕ of the world, while a (hypothetical) observer in opposite time-direction would observe a manifestation of the constituent $\overline{\phi}$ of the antiworld. In this study, however, only inertial observers are considered who live in "our" forward time-direction: all models M_p in $\{M_i\}_{i\in F_1}$ are thus models of the *world*, not of the *antiworld*.

3 Result: the categorical model $\mathscr{C}_{SR}$

3.1 Mathematical definitions: the reference frame of an observer

Definition 3.1 The **pseudo-Riemannian 5-manifold** $(\mathcal{M}, T_{\mathcal{M}}, A_{\mathcal{M}}, T\mathcal{M}, g)$ consists of the set $\mathcal{M} = \mathbb{R}^4 \times \mathbb{U}$, the standard product topology $T_{\mathcal{M}}$, the atlas $A_{\mathcal{M}}$, the tangent bundle $T\mathcal{M}$, and the metric tensor field g, such that

(i) the atlas $A_{\mathcal{M}} = \{(U_t, \phi_t) \mid t \in \mathbb{Q}\}$ contains for every $t \in \mathbb{Q}$ a **chart** (U_t, ϕ_t) given by

$$U_t = (t, t+1) \times \mathbb{R}^3 \times \mathbb{U} \backslash \{\varrho(t)\}$$
$$\phi_t : U_t \to \mathbb{R}^5 \ , \ \phi_t : \begin{cases} (x^0, x^1, x^2, x^3, x^4) \mapsto (x^0, x^1, x^2, x^3, \lfloor t+1 \rfloor + x^4) & if \quad x^4 \in [0, \varrho(t)) \\ (x^0, x^1, x^2, x^3, x^4) \mapsto (x^0, x^1, x^2, x^3, \lfloor t \rfloor + x^4) & if \quad x^4 \in (\varrho(t), 1) \end{cases} \tag{3}$$

where ϱ is the function from Def. 1.4, and $[0, \varrho(t)) = \emptyset$ for $t \in \mathbb{Z}$. See Fig. 1 for an illustration.

(ii) the tangent bundle $T\mathcal{M}$ is the sum set $\bigcup\{T_P(\mathcal{M}) \mid P \in \mathcal{M}\} = \mathcal{M} \times \mathbb{R}^5$, where $T_P(\mathcal{M})$ is the **tangent space** at the point $P \in \mathcal{M}$ given by

$$T_P(\mathcal{M}) = \{P\} \times \mathbb{R}^5 = \{(p^0, p^1, p^2, p^3, p^4, x^0, x^1, x^2, x^3, x^4) \mid x^j \in \mathbb{R}\} \subset \mathbb{R}^{10} \tag{4}$$

If we denote $(p^0, p^1, p^2, p^3, p^4, x^0, x^1, x^2, x^3, x^4) \in T_P(\mathcal{M})$ as $\vec{x}_P$, with $\vec{x}_P = (x^0, x^1, x^2, x^3, x^4)_P$, and if we endow $T_P(\mathcal{M})$ with the operations addition and scalar multiplication given by

$$\vec{x}_P + \vec{y}_P = (x^0 + y^0, x^1 + y^1, x^2 + y^2, x^3 + y^3, x^4 + y^4)_P \tag{5}$$

$$\alpha \cdot \vec{x}_P = (\alpha x^0, \alpha x^1, \alpha x^2, \alpha x^3, \alpha x^4)_P \tag{6}$$

then $(T_P(\mathcal{M}), +, \cdot)$ is isomorphic to the standard five-dimensional vector space $(\mathbb{R}^5, +, \cdot)$.

(iii) the **tensor field** g adds to every point $P \in \mathcal{M}$ a metric tensor $g_P : T_P(\mathcal{M}) \times T_P(\mathcal{M}) \to \mathbb{R}$ such that for any two vectors $\vec{x}_P, \vec{y}_P \in T_P(\mathcal{M})$

$$g_P(\vec{x}_P, \vec{y}_P) = -x^0 y^0 + x^1 y^1 + x^2 y^2 + x^3 y^3 + x^4 y^4 = \eta^{(5)}_{ij} x^i y^j \tag{7}$$

where x^i, y^j are the coordinates with respect to a standard basis of $T_P(\mathcal{M})$.

□

Agreement 3.2 For any 5-tuple $(x^0, ..., x^4)$, a Roman index i, j, k, etc. for the components can take all values from 0 to 4, but a Greek index α, β, etc. can take only nonzero value. So x^j can be any of the components of the 5-tuple $(x^0, ..., x^4)$, while x^α refers only to x^1, x^2, x^3, or x^4. □

Notation 3.3 For any n-tuple $(x^0, ..., x^{n-1})$, the j^{th} component x^j is denoted by $(x^0, ..., x^{n-1})^j$. □

Definition 3.4 The binary operation **scalar multiplication** $\cdot : \mathbb{R} \times \mathcal{M} \to \mathcal{M}$ is for any number $a \in \mathbb{R}$ and any $X = (x^0, x^1, x^2, x^3, x^4) \in \mathcal{M}$ given by

$$a \cdot (x^0, x^1, x^2, x^3, x^4) = (ax^0, ax^1, ax^2, ax^3, a \cdot x^4) \tag{8}$$

where the product $a \cdot x^4$ is given by Def. 1.2.
□

Definition 3.5 The $\mathcal{O}$**-group** $\mathcal{M}$ $(\mathcal{M}, +, \mathcal{O})$ consists of the set $\mathcal{M}$, given in Def. 3.1, the binary operations addition $+ : \mathcal{M} \times \mathcal{M} \to \mathcal{M}$ and the set of operators $\mathcal{O}$, such that

(i) the **sum** of any two elements $X = (x^0, x^1, x^2, x^3, x^4)$ and $Y = (y^0, y^1, y^2, y^3, y^4, y^5)$ of $\mathcal{M}$ is given by

$$(x^0, x^1, x^2, x^3, x^4) + (y^0, y^1, y^2, y^3, y^4) = (x^0 + y^0, x^1 + y^1, x^2 + y^2, x^3 + y^3, x^4 +_1 y^4) \tag{9}$$

where the sum $x^4 +_1 y^4$ is given by Def. 1.2. Note that $(\mathcal{M}, +)$ is thus an abelian group.

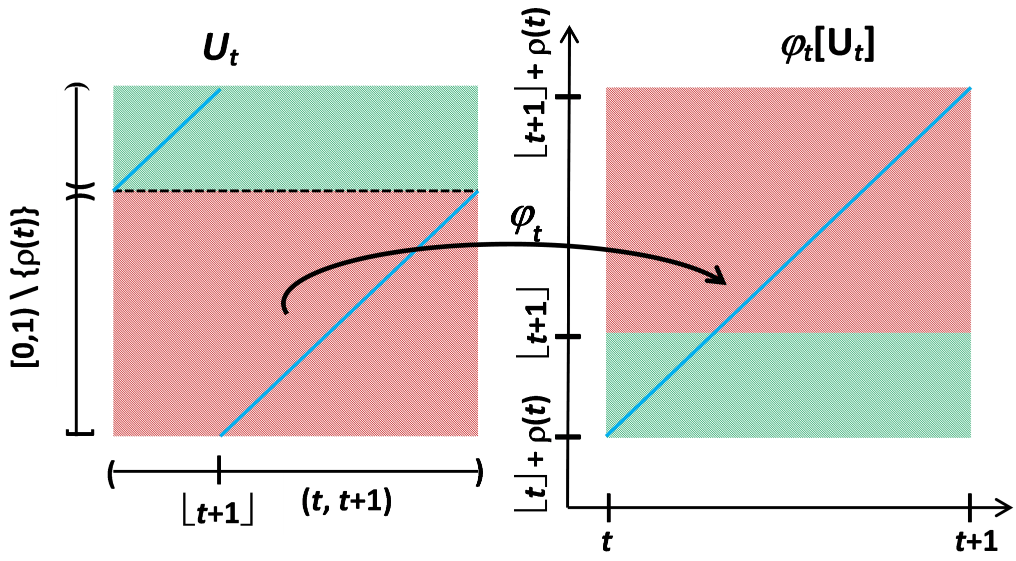

Figure 1: Illustration of the chart (U_t, ϕ_t) in the atlas $A_{\mathcal{M}}$. Suppressing $\mathbb{R}^3$, the square to the left represents $U_t = (t, t+1) \times \mathbb{R}^3 \times \mathbb{U} \backslash \{\varrho(t)\}$; the red part is $(t, t+1) \times [0, \varrho(t))$, the green part is $(t, t+1) \times (\varrho(t), 1)$. The blue line is the line ℓ in $\mathcal{M}$ with parametrization $(u, 0, 0, 0, \varrho(u))$. The square to the right represents $\phi_t[U_t]$, the image of U_t under ϕ_t. The red part, the green part and the blue line in the right square are the images of the corresponding items in the left square. The vertical axis of the right figure shows the total distance traveled in the curled-up dimension since $t = 0$ by an observer moving on ℓ (see Sect. 4.1); if $t = 0$ coincides with the beginning of the universe, then an integer value is the degree of evolution of the EPT that the observer is at.

(ii) for every Lorentz transformation Λ with matrix $\begin{pmatrix} \lambda_{00} & \lambda_{01} & \lambda_{02} & \lambda_{03} \\ \lambda_{10} & \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{20} & \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{30} & \lambda_{31} & \lambda_{32} & \lambda_{33} \end{pmatrix}$ on Minkowski space with signature $(-, +, +, +)$ there is an operator $\lambda \in \mathcal{O}$ with matrix $\begin{pmatrix} \lambda_{00} & \lambda_{01} & \lambda_{02} & \lambda_{03} & 0 \\ \lambda_{10} & \lambda_{11} & \lambda_{12} & \lambda_{13} & 0 \\ \lambda_{20} & \lambda_{21} & \lambda_{22} & \lambda_{23} & 0 \\ \lambda_{30} & \lambda_{31} & \lambda_{32} & \lambda_{33} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ so that

$$\lambda(X) = Y \Leftrightarrow \begin{pmatrix} y^0 \\ y^1 \\ y^2 \\ y^3 \\ y^4 \end{pmatrix} = \begin{pmatrix} \lambda_{00} & \lambda_{01} & \lambda_{02} & \lambda_{03} & 0 \\ \lambda_{10} & \lambda_{11} & \lambda_{12} & \lambda_{13} & 0 \\ \lambda_{20} & \lambda_{21} & \lambda_{22} & \lambda_{23} & 0 \\ \lambda_{30} & \lambda_{31} & \lambda_{32} & \lambda_{33} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \\ x^4 \end{pmatrix} \tag{10}$$

□

Remark 3.6 The space $(\mathcal{M}, +, \cdot)$ is **not** a vector space, for the same reason as mentioned in the proof of Cor. 1.3: it is **not** for any $\alpha \in \mathbb{R}$ and $X, Y \in \mathcal{M}$ the case that $\alpha \cdot (X + Y) = \alpha \cdot X + \alpha \cdot Y$. However, for any operator $\lambda \in \mathcal{O}$ and $X, Y \in \mathcal{M}$ we **do** get that $\lambda(X + Y) = \lambda(X) + \lambda(Y)$. □

Definition 3.7 A **null path** is a curve $\mathcal{C} \subset \mathcal{M}$ parameterized by $u \in \mathbb{R}$, such that the tangent vector $\vec{v}(u)_{X(u)} \in T_{X(u)}(\mathcal{M})$ of the path $\mathcal{C}$ at the point $X(u) \in \mathcal{M}$ satisfies

$$g_{X(u)}(\vec{v}(u)_{X(u)}, \vec{v}(u)_{X(u)}) = 0 \tag{11}$$

at any point $X(u)$ on the curve $\mathcal{C}$. □

Corollary 3.8 Let $\mathcal{C}$ be any null path in $\mathcal{M}$, and let $\lambda \in \mathcal{O}$. Then $\lambda[\mathcal{C}]$ is a null path.

Proof The proof is omitted. □

Agreement 3.9 (Units) In the remainder of this text we will use **Planck units**: both Planck length and Planck time are scaled to 1. Furthermore, for the sake of simplicity we will use rectangular coordinates so that we can use the components $\eta_{ij}^{(5)}$ of the metric tensor $\eta^{(5)}$ of Eq. (7). □

Definition 3.10 (5D IRF) The **reference frame of an inertial observer in five-dimensional spacetime** is the manifold $\mathcal{M}$ of Def. 3.1. Such an inertial reference frame will henceforth be referred to by the acronym '5D IRF'. For a point $X = (x^0, x^1, x^2, x^3, x^4) \in \mathcal{M}$, the real number x^0 is the time coordinate, the three real numbers x^1, x^2, x^3 are the "regular" spatial coordinates, and the real number x^4 represents the coordinate in the curled-up fifth dimension. We will call this number $x^4 \in \mathbb{U}$ simply the *fourth spatial coordinate*; alternatively it can be called a *degree* as it is (as we will see) the degree of completion of an individual process. □

Remark 3.11 (Presupposition) Def. 3.10 thus implies that the present categorical model of the EPT only applies for inertial observers: it is, thus, a presupposition that all observers are inertial observers. □

Remark 3.12 (Dimensionality) In broad lines, in the universe of the EPT the observable process of evolution can be indexed by a parameter called *degrees of evolution*: at each integer-valued degree of evolution n there are a finite number $\omega(n)$ of individual processes from that degree of evolution to the next. The EPT is then a description of what happens in the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution: the idea is that this is a generic description that applies to all individual processes—that is, the same principles apply for any value of k and n.

The point now is that every individual process involves a unit displacement in degrees of evolution: the compact fifth dimension of $\mathcal{M}$ is then a **model of the set of physically distinct points** in the spatial dimension formed by the degrees of evolution—a displacement in degrees of evolution is thus a displacement in *an additional spatial dimension*. The set $\mathbb{U}$ is then the set of natural coordinates in this dimension.[2]

Thus speaking, the dimension of degrees of evolution can be modeled by the set $\mathbb{R}$ in Planck units together with an equivalence relation $\sim$ given by

$$x \sim y \Leftrightarrow x \equiv y \pmod 1 \tag{12}$$

where $x \sim y$ is to be interpreted as 'x and y are **physically** the same point'. As there is a natural bijection $\phi : [x]_\sim \leftrightarrow x$ between the cells $[x]_\sim$ of the partitioning of $\mathbb{R}$ induced by $\sim$ and the elements $x \in \mathbb{U}$, the set of physically distinct points in the dimension of degrees of evolution can thus be modeled in a natural way by the set $\mathbb{U} = [0, 1)$. The manifold $\mathcal{M}$ is then the union of this curled-up dimension with Minkowski spacetime.

To be clear: a degree of evolution n can take any value in $\mathbb{R}$, but we have $n \in [x]_\sim$ for precisely one of the cells $[x]_\sim \subset \mathbb{R}$, and thus n corresponds with the fourth spatial coordinate $\varrho(x)$. But for an observer who has set his clocks at $t = 0$ at the beginning of the universe, for a particle traveling on the curve $(t, 0, 0, 0, \varrho(t))$ the chart (U_t, ϕ_t) reveals at which degree of evolution the particle finds itself at any point in time t; cf. Fig. 1. □

3.2 Mathematical definitions: the state postulate

The ultimate building blocks of the universe of the EPT are called 'phase quanta'. Each such phase quantum is designated (i.e., referred to) by a symbol in the EPT: we must therefore distinguish between the *material object*, i.e. the thing in the physical world that is referred to, and the *formal object*, the thing in the mathematical universe that refers to a material object—the *interpretation rules* of the EPT thus dictate which formal object refers to which material object. As mentioned in Sect. 2, in the EPT these formal objects are abstract sets, that is, sets whose elements are not specified: the EPT thus states elementary principles without reference to any coordinate system.

In a set-theoretic model of the EPT, a formal object ϕ that refers to a material object is interpreted in a concrete set-theoretical domain D, such that its interpretation $I(\phi)$ is a representation of the state of the material object designated by ϕ in the reference frame of an observer. The objective of this section is to develop a state postulate, in which it will be laid down what the set-theoretical domain D is and what the general form is of the formal objects representing the state of a phase quantum in the reference frame of an observer.

[2] The necessity of the compactness of this dimension can be illustrated with the simplest of examples. Consider two observers at rest across each other. Photons emitted by both observers remain at a hyperplane at constant degree of evolution, while the observers themselves propagate through the dimension of degrees of evolution. So if this wouldn't be curled-up, the observers could never see each other.

The problem to be tackled in this section is that a function f is required that can represent the distribution of energy over isolated points of a spatial dimension of the manifold $\mathcal{M}$ of Def. 3.1. That is, the problem is that a function f is required that must satisfy the following two conditions:

$$\begin{array}{ll} \text{(i)} & f \in R^{\mathbb{R}} \\ \text{(ii)} & \left\{ \begin{array}{l} f(x) = 0 \Leftrightarrow x \neq 0 \\ \int_{-\infty}^{+\infty} f(x)dx = 1 \end{array} \right. \end{array} \tag{13}$$

Here R is a number ring. Our attention is then drawn to the Dirac delta, denoted by the symbol δ, which is defined to have the following properties for real numbers x [14]:

$$\left\{ \begin{array}{l} \delta(x) = 0 \Leftrightarrow x \neq 0 \\ \int_{-\infty}^{+\infty} \delta(x)dx = 1 \end{array} \right. \tag{14}$$

A standard definition of δ is as a limit of a sequence of real functions [14]: let, for positive integers n, the function $f_n : \mathbb{R} \to \mathbb{R}$ be given by $f_n(x) = n$ for $x \in (-\frac{1}{2n}, \frac{1}{2n})$ and $f_n(x) = 0$ else; then

$$\delta := \lim_{n \to \infty} f_n \tag{15}$$

But this limit does not exist in the space of all functions from $\mathbb{R}$ to $\mathbb{R}$. That is, because the value of δ at $x = 0$ is not finite, the Dirac delta defined by Eq. (15) is not an ordinary function on the reals: this definition does, therefore, not satisfy the clause (i) of Eq. (13). However, the real number field can be extended to the hyperreal number field $^*\mathbb{R}$ [15]. This allows an identification of δ with a hyperreal function $^*\delta : \ ^*\mathbb{R} \to \ ^*\mathbb{R}$ given by

$$\left\{ \begin{array}{l} ^*\delta(x) = 0 \Leftrightarrow x \notin [-\frac{dx}{2}, \frac{dx}{2}] \\ ^*\delta(x) = \frac{1}{dx} \Leftrightarrow x \in [-\frac{dx}{2}, \frac{dx}{2}] \end{array} \right. \tag{16}$$

where 'dx' is an infinitesimal hyperreal number [16]. However, an identification $\delta := \ ^*\delta$ defines the Dirac delta δ as an element of the function space $^*\mathbb{R}^{^*\mathbb{R}}$: this definition thus also violates clause (i) of Eq. (13). That being said, below we identify the Dirac delta δ with an ordinary function f with domain $\mathbb{R}$ as a first step towards the aim stated at the beginning of this section.

For our present purposes we need hyperreal numbers, but we do not need *all* of the hyperreal number field. So first of all we apply Ockham's razor and we define the ordered ring of the expanded reals as the part of the hyperreals that we need:

Definition 3.13 *The* ***ordered ring of expanded real numbers*** *is the subring* $(^*_+\mathbb{R}, +\ ,\ \cdot\ , >)$ *of the hyperreal number field* $(^*\mathbb{R}, +\ ,\ \cdot\ , >)$ *given by*

$$^*_+\mathbb{R} = \{\xi \in \ ^*\mathbb{R} \mid \xi = a_1\omega^{p_1} + a_2\omega^{p_2} + \ldots + a_n\omega^{p_n}, n \in \mathbb{N}^+, p_1 > p_2 > \ldots > p_n \geq 0, a_j \in \mathbb{R}\} \tag{17}$$

That is, the set $^*_+\mathbb{R}$ *contains the real numbers and, as indicated by the left subscript '+', those hyperreal numbers* $\xi \notin \mathbb{R}$ *in which a finite number of* positive *powers of the infinitely big hyperreal number* ω *with* $|\omega| = \infty$ *occur. As a set, (non-real) expanded real numbers can, for* <u>nonzero</u> a_j*, be defined according to*

$$a_1\omega := \{\langle a_1, 0\rangle, \langle a_1, 1\rangle, \langle a_1, 2\rangle, \ldots\} \tag{18}$$

$$\sum_{j=1}^{n} a_j\omega^{p_j} := a_1\omega^{p_1} \cup \{\sum_{j=2}^{n} a_j\omega^{p_j}\} = \{\sum_{j=2}^{n} a_j\omega^{p_j}, \langle a_1\omega^{p_1-1}, 0\rangle, \langle a_1\omega^{p_1-1}, 1\rangle, \langle a_1\omega^{p_1-1}, 2\rangle, \ldots\} \tag{19}$$

with $n \geq 2$ *in Eq. (19) and* $p_1 > p_2 > \ldots > p_n \geq 0$ *as in Eq. (17); the nonzero real* a_j*'s can be represented by Dedekind cuts. Note that an inequality* $a_1\omega + a_2 \neq a_1\omega$ *is then an inequality of sets.* □

Agreement 3.14 In the remainder of this text we will take the notation $\sum_{j=1}^{n} a_j\omega^{p_j}$ for an expanded real number $x \in \ ^*_+\mathbb{R}$ to *imply* that $p_1 > p_2 > \ldots > p_n \geq 0$ as in Def. 3.13. □

The space of all functions $f : \mathbb{R} \to \ ^*_+\mathbb{R}$ then forms a vector algebra over $\mathbb{R}$, when function addition, scalar multiplication, and function multiplication are defined naturally, so

$$(f + g)(x) = f(x) + g(x) \tag{20}$$

$$(\alpha \cdot f)(x) = \alpha f(x) \tag{21}$$

$$(f \cdot g)(x) = f(x)g(x) \tag{22}$$

In this function space $^*_+\mathbb{R}^{\mathbb{R}}$ we now define the expanded real delta functions $\alpha\delta(x - \beta)$ as follows:

Definition 3.15 For any $\alpha, \beta \in \mathbb{R}$, the **expanded real delta function** $\alpha\delta(x-\beta) : \mathbb{R} \to {}^*_+\mathbb{R}$ is given by

(i) $x \neq \beta \Rightarrow \alpha\delta(x-\beta) = 0$

(ii) $x = \beta \Rightarrow \alpha\delta(x-\beta) = \alpha\omega$

□

The next step is then to define how to integrate an arbitrary function $f : \mathbb{R} \to {}^*_+\mathbb{R}$. For that matter, the following two definitions are useful.

Definition 3.16 *Let $x = \sum_{j=1}^{n} a_j\omega^{p_j}$ be an expanded real number. The **real part** of x is then the number $Re(x)$ given by*

$$\begin{cases} p_n = 0 \Rightarrow Re(x) = a_n \\ p_n > 0 \Rightarrow Re(x) = 0 \end{cases} \tag{23}$$

*Likewise, the **hyperreal part** of x is then the number $Hy(x)$ given by*

$$\begin{cases} p_n = 0 \Rightarrow Hy(x) = x - a_n \\ p_n > 0 \Rightarrow Hy(x) = x \end{cases} \tag{24}$$

So, for any expanded real number x we have $x = Re(x) + Hy(x)$. □

Definition 3.17 *Let $f : \mathbb{R} \to {}^*_+\mathbb{R}$. Then the **real part of** f is the function $f_{Re} : \mathbb{R} \to {}^*_+\mathbb{R}$ given by*

$$f_{Re} : x \mapsto Re(f(x)) \tag{25}$$

*Likewise the **hyperreal part of** f is the function $f_{Hy} : \mathbb{R} \to {}^*_+\mathbb{R}$ given by*

$$f_{Hy} : x \mapsto Hy(f(x)) \tag{26}$$

Thus speaking, for any $f : \mathbb{R} \to {}^*_+\mathbb{R}$ we have $f = f_{Re} + f_{Hy}$. □

Note that for $f = \alpha\delta(x-\beta)$ we have $f = f_{Hy}$. We can now define the integral over $\mathbb{R}$ of an arbitrary expanded real function f:

Definition 3.18 *Let $\mathcal{R}^1(\mathbb{R})$ be the set of Riemann integrable functions on $\mathbb{R}$, and let the set of all integrable expanded real functions on $\mathbb{R}$ be denoted by ${}^*_+\mathcal{R}^1(\mathbb{R})$. Let $f \in {}^*_+\mathbb{R}^{\mathbb{R}}$; then $f \in {}^*_+\mathcal{R}^1(\mathbb{R})$ **if and only if***

$$f_{Re} \in \mathcal{R}^1(\mathbb{R}) \tag{27}$$

$$f_{Hy} = \sum_{n=1}^{\infty} \alpha_n\delta(x - \beta_n) \tag{28}$$

for some convergent series $\sum_{n=1}^{\infty} \alpha_n = s \in \mathbb{R}$. Furthermore, if $f \in {}^*_+\mathcal{R}^1(\mathbb{R})$ then

$$\int_{-\infty}^{+\infty} f(x)dx = \int_{-\infty}^{+\infty} f_{Re}(x)dx + \int_{-\infty}^{+\infty} f_{Hy}(x)dx = \int_{-\infty}^{+\infty} f_{Re}(x)dx + \sum_{n=1}^{\infty} \alpha_n \tag{29}$$

□

Def. 3.18 thus says that an expanded real function f on $\mathbb{R}$ is integrable if and only if the real part of f is Riemann integrable and the hyperreal part of f is a countable sum of expanded real delta functions with the coefficients forming a convergent series. A corollary of Def. 3.18 is that for the integral of the expanded real function $\alpha\delta(x-\beta)$ over $\mathbb{R}$ we have

$$\int_{-\infty}^{+\infty} \alpha\delta(x-\beta)dx := \alpha \tag{30}$$

The expanded real delta function $1\delta(x-0)$ has then the desired properties of the Dirac delta δ displayed in Eq. (14). Thus speaking, if we identify δ with $1\delta(x-0)$, then the Dirac delta as a set simply becomes the graph of $1\delta(x-0)$:

$$\delta := \{\langle x, \xi\rangle \in \mathbb{R} \times^*_+ \mathbb{R} \mid x \neq 0 \Rightarrow \xi = 0 \wedge x = 0 \Rightarrow \xi = \omega\} \tag{31}$$

The identification $\delta := 1(\delta(x-0)$ thus yields a definition of the Dirac delta δ as an ordinary function $1\delta(x-0)$ with domain $\mathbb{R}$ that can be used to construct functions f satisfying Eq. (13).

Def. 3.18 can be generalized to integrable expanded real functions on $\mathbb{R}^n$: a function $f : \mathbb{R}^n \to {}^*_+\mathbb{R}$ is then integrable if and only if it is the sum of a Riemann integrable real function f_{Re} on $\mathbb{R}^n$ and a countable sum $\sum_{j=1}^{\infty} f_{j,Hy}$ of hyperreal functions $f_{j,Hy}$ on $\mathbb{R}^n$ that are products of a function g_j of $n-1$ variables and h_j of one variable, such that g_j is associated with a function in ${}^*_+\mathcal{R}^1(\mathbb{R}^{n-1})$ and h_j with a function in ${}^*_+\mathcal{R}^1(\mathbb{R})$. For $n = 3$, for example, we can consider the function $f : \mathbb{R}^3 \to {}^*_+\mathbb{R}$ for which $f : (x, y, z) \mapsto \alpha\delta(x - \beta_x)\delta(y - \beta_y)\delta(z - \beta_z)$; for the integral over $\mathbb{R}^3$ we then have

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x, y, z)dxdydz = \alpha \tag{32}$$

A formal definition is omitted.We can now return to the aim of this section stated in the beginning and formulate the *state postulate*:

Postulate 3.19 In the categorical model $\mathscr{C}_{SR}$ of the EPT, the state of a phase quantum in the 5D IRF of an observer $\mathcal{O}$ is represented by a function $f : \mathcal{M} \to {}^*_+\mathbb{R}$ for which

$$f : (t, x, y, z, n) \mapsto E \cdot \chi(t, n)\delta(x - r^1(t))\delta(y - r^2(t))\delta(z - r^3(t)) \tag{33}$$

where E is the energy of the state and $\chi : \mathcal{M} \to {}^*_+\mathbb{R}$ is a characteristic function having the value 0 at times t when the state doesn't exist, and the value 1 at times t when the state exists with

$$\chi(t, n) = \chi(t, n') = 1 \Rightarrow n = n' \tag{34}$$

That is, at every time t that the state exists it occurs only at one degree n, and the energy E of the state is then (i.e. at the time t) distributed over the one point $(t, r^1(t), r^2(t), r^3(t), n) \in \mathcal{M}$. □

Recall that the EPT is not a quantum theory, so in the present categorical model of the EPT the above state postulate is to be viewed as an equivalent of e.g. the state postulate of quantum mechanics, which states that a quantum state is represented by an element ψ of a Hilbert space $\mathscr{H}$ with norm $\|\psi\| = 1$—this goes back to Schrödinger's early works, e.g. [17]. Similarly, here we have that the state of a phase quantum is represented by an element f of the function space ${}^*_+\mathbb{R}^{\mathcal{M}}$ for which

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(t, x, y, z, n)dxdydz = E \tag{35}$$

for constant t, n with $\chi(t, n) = 1$. In the next section, set-theoretic models of the EPT are specified in accordance with this state postulate. So to emphasize it: while it is useful to identify the Dirac delta with a generalized function for applications in differential equations, in this section we have identified the Dirac delta with an integrable hyperreal function on the real numbers because that is useful for application to set-theoretic models of the EPT.

3.3 The objects of the category $\mathscr{C}_{SR}$

Below a generic set-theoretic model $M_{\mathbb{Z},\omega,\mathcal{O}}$ of the EPT is specified in a number of *interpretations* along the lines of the state postulate 3.19. In this model, the set of all integer-valued degrees of evolution is modeled by $\mathbb{Z}$, and the number of individual processes from any integer-valued degree of evolution n to the next is ω: this is a generic constant which does not depend on n. Correspondingly, the set S_ω is the section of positive integers up to and including ω:

$$S_\omega := \{1, 2, \ldots, \omega\} \tag{36}$$

For the constant k in the k^{th} process from the n^{th} to the $(n + 1)^{\text{th}}$ degree of evolution we thus have $k \in S_\omega$. Furthermore, the interpretations make use of the following notation and definition:

Notation 3.20 Let $M_{\mathbb{Z},\omega,\mathcal{O}}$ be a set-theoretic model of the EPT; let $I_{\mathbb{Z},\omega,\mathcal{O}}$ be the interpretation function that maps any constant ϕ of the EPT to its interpretation $I_{\mathbb{Z},\omega,\mathcal{O}}(\phi)$ in the language of $M_{\mathbb{Z},\omega,\mathcal{O}}$. For a constant ϕ of the EPT referring to a phase quantum, the expression

$$\phi \xrightarrow{\mathcal{O}} f \tag{37}$$

is then a notation for $I_{\mathbb{Z},\omega,\mathcal{O}}(\phi) = f$, and has to be read as: 'the state of the phase quantum, designated by ϕ, in the coordinate system of the observer $\mathcal{O}$ is represented by f'. □

To specify the generic set-theoretic model $M_{\mathbb{Z},\omega,\mathcal{O}}$ of the EPT, we must begin by defining the set of *monads*. A 'monad' in the EPT is an abstraction of an indivisible massive particle: in this model, a monadic state is an indivisible building block of the world as seen by observer $\mathcal{O}$—its properties then relate to the properties of the monad defined below.

Definition 3.21 (Monads) Let $M_{\mathbb{Z},\omega,\mathcal{O}}$ be a set-theoretic model of the EPT. The **set of all monads** in $M_{\mathbb{Z},\omega,\mathcal{O}}$ is then the set

$$A_{\omega,\mathcal{O}} = \{\langle k, \sigma_k, \chi_k\rangle \mid k \in S_\omega\} \tag{38}$$

For any $k \in S_\omega$, the three-tuple $\langle k, \sigma_k, \chi_k\rangle \in S$ is the **k$^{\text{th}}$ monad**; the constant σ_k is the **rest mass spectrum** of the k^{th} monad; the constant $\chi_k \in \{-1, 1\}$ is the **characteristic number of normality** of the k^{th} monad. In this model, the rest mass spectrum is a constant function

$$\sigma_k : \mathbb{Z} \to \mathbb{R}, \sigma_k : n \to m_k \tag{39}$$

that adds the number $m_k > 0$, the rest mass of the k^{th} monad, to a degree of evolution n. □

Before specifying the interpretations of constants and axioms of the EPT, recall that the support of a function f is defined as follows [18]:

Definition 3.22 Let X be any nonempty set, let V be a vector space and let f be a function $f : X \to V$; then the **support of f**, notation: supp f, is the subset of X made up of precisely those elements that have a nonzero function value:

$$\text{supp } f = \{x \in X \mid f(x) \neq 0\} \tag{40}$$

□

Interpretation 3.23 For integers $n \in \mathbb{Z}$ and $k \in S_\omega$, the constant ${}^{EP}\mu_k^n$ of the EPT designates the *extended particlelike matter quantum at the n^{th} degree of evolution associated to the k^{th} monad.* In the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ we then have

$$ {}^{EP}\mu_k^n \xrightarrow{\mathcal{O}} s_k^n \tag{41}$$

$$ s_k^n : \mathcal{M} \to {}^*_+\mathbb{R} \tag{42}$$

$$ \text{supp } s_k^n = \{(t_{n,k}, x_{n,k}, y_{n,k}, z_{n,k}, 0)\} = \{X_{n,k}\} \tag{43}$$

$$ s_k^n : (t, x, y, z, u) \mapsto E_{n,k}^{EP} \cdot \chi_{n,k}^{EP}(t, u)\delta(x - x_{n,k})\delta(y - y_{n,k})\delta(z - z_{n,k}) \tag{44}$$

Thus speaking, the state of the particlelike matter quantum, designated by the symbol ${}^{EP}\mu_k^n$ in the EPT, in the coordinate system of the observer $\mathcal{O}$ is modeled as a **point-particle** with energy $E = E_{n,k}^{EP} > 0$ represented by the above function $s_k^n \in {}^*_+\mathbb{R}^{\mathcal{M}}$. Note that the point-particle only exists at the one spatiotemporal position $X_{n,k}$ in the 5D IRF of $\mathcal{O}$, so $\chi_{n,k}^{EP}(t, u) = 1$ if $(t, u) = (t_{n,k}, 0)$ and $\chi_{n,k}^{EP}(t, u) = 0$ else. □

Agreement 3.24 We will henceforth refer to the state represented by the function s_k^n as the 'particle state of the k^{th} monad at the n^{th} degree of evolution in the 5D IRF of the observer $\mathcal{O}$'. □

Now that we have the monadic particle states, we are going to let these evolve according to the principles of the EPT, which are formulated in terms of *phase quanta*: the idea for this model is that the particlelike state of the k^{th} monad at the n^{th} degree of evolution is the initial state at the start of the k^{th} process from the n^{th} to the $(n + 1)^{\text{th}}$ degree of evolution. So we first interpret the constants of the EPT referring to phase quanta, and then we interpret the principles of the EPT.

Interpretation 3.25 For integers $n \in \mathbb{Z}$ and $k \in S_\omega$, the constant ${}^{EP}\Phi_k^n$ of the EPT designates the *extended particlelike phase quantum occurring in the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution.* In the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ we then have

$$ {}^{EP}\Phi_k^n \xrightarrow{\mathcal{O}} {}^{EP}f_k^n \tag{45}$$

$$ {}^{EP}f_k^n : \mathcal{M} \to {}^*_+\mathbb{R} \ , \ {}^{EP}f_k^n = s_k^n \tag{46}$$

□

Thus speaking, in $M_{\mathbb{Z},\omega,\mathcal{O}}$ the state of the phase quantum, designated by the symbol ${}^{EP}\Phi^n_k$ in the EPT, in the 5D IRF of the observer $\mathcal{O}$ is the particle state of the k^{th} monad at the n^{th} degree of evolution in the 5D IRF of the observer $\mathcal{O}$. Thus speaking, in the 5D IRF of the observer $\mathcal{O}$, the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution starts with a point-particle with energy $E^{EP}_{n,k}$ at spatiotemporal position $X_{n,k}$. Moreover, Int. 3.25 associates the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution with the k^{th} monad: the properties of the monad defined in Def. 3.21 will thus occur in the said process.

Remark 3.26 To emphasize it: in a more elaborate model of the EPT the phase quantum ${}^{EP}\Phi^n_k$ will be modeled as an *aggregation* of monadic particle states, and these do not have to be point-particles. Thus speaking, Int. 3.25 forces us to treat, for example, a deuterium nucleus as a monadic state—although we already know that it is composed of a neutron and a proton. The crux here is that we are only interested in showing that the EPT agrees with SR: therefore, we keep the internal states of massive particles as simple as possible—that is, all massive particles are modeled as elementary point-particles. □

Interpretation 3.27 For integers $n \in \mathbb{Z}$ and $k \in S_\omega$, the constant ${}^{NW}\Phi^n_k$ of the EPT designates the *non-local wavelike phase quantum occurring in the* k^{th} *process from the* n^{th} *to the* $(n+1)^{\text{th}}$ *degree of evolution.* In the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ we then have

$${}^{NW}\Phi^n_k \xrightarrow{\mathcal{O}} {}^{NW}f^n_k \tag{47}$$

$${}^{NW}f^n_k : \mathcal{M} \to^*_+ \mathbb{R} \tag{48}$$

$$\text{supp } {}^{NW}f^n_k = \overline{\Delta X}_{n,k} \tag{49}$$

Here $\overline{\Delta X}_{n,k}$ is a line segment in the 5D IRF of the observer $\mathcal{O}$ determined by the spatiotemporal position $X_{n,k}$ of Int. 3.23 and a displacement $(\Delta t_{n,k}, \Delta x_{n,k}, \Delta y_{n,k}, \Delta z_{n,k})$ in 4D spacetime with $\Delta t_{n,k} > 0$:

$$\overline{\Delta X}_{n,k} : \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \\ x^4 \end{pmatrix} = \begin{pmatrix} t_{n,k} \\ x_{n,k} \\ y_{n,k} \\ z_{n,k} \\ 0 \end{pmatrix} + \begin{pmatrix} \lambda \cdot \Delta t_{n,k} \\ \lambda \cdot \Delta x_{n,k} \\ \lambda \cdot \Delta y_{n,k} \\ \lambda \cdot \Delta z_{n,k} \\ \varrho(\chi_k \cdot \lambda) \end{pmatrix} \quad , \quad \lambda \in (0,1) \tag{50}$$

where χ_k is the characteristic number of normality of the k^{th} monad given in Def. 3.21, and ϱ is again the function from Def. 1.4. For $t \in (t_{n,k}, t_{n,k} + \Delta t_{n,k})$ we have for the function value

$${}^{NW}f^n_k : (t,x,y,z,u) \mapsto E^{NW}_{n,k} \cdot \chi^{NW}_{n,k}(t,u)\delta(x - x^1(t))\delta(y - x^2(t))\delta(z - x^3(t)) \tag{51}$$

with $x^1(t) = x_{n,k} + (t - t_{n,k})\frac{\Delta x_{n,k}}{\Delta t_{n,k}}$, $x^2(t) = y_{n,k} + (t - t_{n,k})\frac{\Delta y_{n,k}}{\Delta t_{n,k}}$, $x^3(t) = z_{n,k} + (t - t_{n,k})\frac{\Delta z_{n,k}}{\Delta t_{n,k}}$, and $\chi^{NW}_{n,k}(t,u) = 1$ if $(t,u) = (t_{n,k} + \lambda\Delta t_{n,k}, \varrho(\chi_k \cdot \lambda))$ for some $\lambda \in (0,1)$ and $\chi^{NW}_{n,k}(t,u) = 0$ else; for $t \notin (t_{n,k}, t_{n,k} + \Delta t_{n,k})$ we have for the function value ${}^{NW}f^n_k(t,x,y,z,u) = 0$ everywhere. Thus speaking, the state of the phase quantum, designated by the symbol ${}^{NW}\Phi^n_k$ in the EPT, in the 5D IRF of the observer $\mathcal{O}$ is a **time-like string** with energy $E = E^{NW}_{n,k} > 0$ and spatiotemporal extension $\overline{\Delta X}_{n,k}$, represented by the above function ${}^{NW}f^n_k \in {}^*_+\mathbb{R}^{\mathcal{M}}$. At every point $X(\lambda)$ of its spatiotemporal extension (with the above parametrization), the time-like string is associated with a **5-momentum** $\vec{p}^{(5)}_{X(\lambda)} \in T_{X(\lambda)}(\mathcal{M})$ for which

$$\vec{p}^{(5)}_{X(\lambda)} = m_k \cdot (\frac{dx^0}{d\lambda}, \frac{dx^1}{d\lambda}, \frac{dx^2}{d\lambda}, \frac{dx^3}{d\lambda}, \frac{dx^4}{d\lambda})_{X(\lambda)} = (E^{NW}_{n,k}, p^1_{n,k}, p^2_{n,k}, p^3_{n,k}, \chi_k \cdot m_k)_{X(\lambda)} \tag{52}$$

$$g_{X(\lambda)}(\vec{p}^{(5)}_{X(\lambda)}, \vec{p}^{(5)}_{X(\lambda)}) = -(E^{NW}_{n,k})^2 + (p^1_{n,k})^2 + (p^2_{n,k})^2 + (p^3_{n,k})^2 + (m_k)^2 = 0 \tag{53}$$

where m_k in Eq. (52) is the rest mass of the k^{th} monad as given by Def. 3.21. □

Note that the components $p^j_{n,k}$ of $\vec{p}^{(5)}_{X(\lambda)}$ in Eq. (52) are numbers, so $\frac{d^2x^j}{d\lambda^2} = 0$. We can view the time-like string ${}^{NW}f^n_k$ as a *wave traveling in a straight line*, associated with energy $E^{NW}_{n,k}$ and **constant** spatial momenta $p^\alpha_{n,k}$.

Interpretation 3.28 For integers $n \in \mathbb{Z}$ and $k \in S_\omega$, the constant ${}^{NP}\Phi_k^{n+1}$ of the EPT designates the *non-extended particlelike phase quantum occurring in the* k^{th} *process from the* n^{th} *to the* $(n+1)^{\text{th}}$ *degree of evolution.* In the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ we then have

$${}^{NP}\Phi_k^{n+1} \xrightarrow{\mathcal{O}} {}^{NP}f_k^{n+1} \tag{54}$$

$${}^{NP}f_k^{n+1} : \mathcal{M} \to {}^*_+\mathbb{R} \tag{55}$$

$$\text{supp } {}^{NP}f_k^{n+1} = \{(t_{n+1,k}, x_{n+1,k}, y_{n+1,k}, z_{n+1,k}, 0)\} = \{X_{n+1,k}\} \quad , \quad t_{n+1,k} = t_{n,k} + \Delta t_{n,k} \tag{56}$$

$${}^{NP}f_k^{n+1} : (t, x, y, z, u) \mapsto E_{n+1,k}^{NP} \chi_{n+1,k}^{NP}(t, u)\delta(x - x_{n+1,k})\delta(y - y_{n+1,k})\delta(z - z_{n+1,k}) \tag{57}$$

Thus speaking, in $M_{\mathbb{Z},\omega,\mathcal{O}}$ the state of the phase quantum, designated by the symbol ${}^{NP}\Phi_k^{n+1}$ in the EPT, in the 5D IRF of the observer $\mathcal{O}$ is modeled by a **point-particle** with energy $E = E_{n+1,k}^{NP} > 0$ represented by the above function ${}^{NP}f_k^{n+1} \in {}^*_+\mathbb{R}^{\mathcal{M}}$. Note that the point-particle only exists at the one spatiotemporal position $X_{n+1,k}$ in the 5D IRF of $\mathcal{O}$, so $\chi_{n+1,k}^{NP}(t, u) = 1$ if $(t, u) = (t_{n+1,k}, 0)$ and $\chi_{n+1,k}^{NP}(t, u) = 0$ else. □

Interpretation 3.29 For integers $n \in \mathbb{Z}$ and $k \in S_\omega$, the constant ${}^{LW}\Phi_k^{n+1}$ of the EPT designates the *local wavelike phase quantum occurring in the* k^{th} *process from the* n^{th} *to the* $(n+1)^{\text{th}}$ *degree of evolution.* In the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ we then have

$${}^{LW}\Phi_k^{n+1} \xrightarrow{\mathcal{O}} \gamma_k^{n+1} \tag{58}$$

$$\gamma_k^{n+1} : \mathcal{M} \to {}^*_+\mathbb{R} \tag{59}$$

$$\text{supp } \gamma_k^{n+1} = \ell_{n+1,k}^\gamma \tag{60}$$

Here $\ell_{n+1,k}^\gamma \subset \mathcal{M}$ is a line segment in the 5D IRF of the observer $\mathcal{O}$ determined by the spatiotemporal position $X_{n+1,k}$ of Int. 3.28 and an element $(1, v^1, v^2, v^3, 0) \in \mathcal{M}$:

$$\ell_{n+1,k}^\gamma : \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \\ x^4 \end{pmatrix} = \begin{pmatrix} t_{n+1,k} \\ x_{n+1,k} \\ y_{n+1,k} \\ z_{n+1,k} \\ 0 \end{pmatrix} + \mu \cdot \begin{pmatrix} 1 \\ v^1 \\ v^2 \\ v^3 \\ 0 \end{pmatrix} \quad , \quad \mu \in (0, t_{\text{end}}) \tag{61}$$

For $t \notin (t_{n+1,k}, t_{n+1,k} + t_{\text{end}})$ we have $\gamma_k^{n+1}(t, n, x, y, z, u) = 0$ everywhere, but for $t \in (t_{n+1,k}, t_{n+1,k} + t_{\text{end}})$ we have for the function value

$$\gamma_k^{n+1} : (t, n, x, y, z, u) \mapsto \Delta E_{n+1,k} \cdot \chi_{n+1,k}^{LW}(t, u)\delta(x - x^1(t))\delta(y - x^2(t))\delta(z - x^3(t)) \tag{62}$$

where $x^1(t) = (t - t_{n+1,k})v^1$, $x^2(t) = (t - t_{n+1,k})v^2$, $x^1(t) = (t - t_{n+1,k})v^3$, and $\chi_{n+1,k}^{LW} : M \to {}^*_+\mathbb{R}$ is a characteristic function with $\chi_{n+1,k}^{LW}(t, u) = 1$ if $u = 0$ and $\chi_{n+1,k}^{LW}(t, u) = 0$ else. Thus speaking, in $M_{\mathbb{Z},\omega,\mathcal{O}}$ the state of the phase quantum, designated by the symbol ${}^{LW}\Phi_k^{n+1}$ in the EPT, in the 5D IRF of the observer $\mathcal{O}$ is modeled by a **γ-ray** with path $\ell_{n+1,k}^\gamma$ and with energy $E = \Delta E_{n+1,k}^{NP} > 0$, represented by the above function ${}^{NP}f_k^{n+1} \in {}^*_+\mathbb{R}^{\mathcal{M}}$. If the γ-ray gets absorbed at a time $t > t_{n+1,k}$, then t_{end} has the finite value t; if no absorption takes place, then in Eq. (61) we have $(0, t_{\text{end}}) = (0, \infty)$. At every point $X(\mu)$ of its path (with the above parametrization), the γ-ray is associated with a **5-momentum** $\Delta\vec{p}^{(5)}_{X(\mu)} \in T_{X(\mu)}(\mathcal{M})$ for which

$$\Delta\vec{p}^{(5)}_{X(\mu)} = \Delta E_{n+1,k} \cdot \left(\frac{dx^0}{d\mu}, \frac{dx^1}{d\mu}, \frac{dx^2}{d\mu}, \frac{dx^3}{d\mu}, \frac{dx^4}{d\mu}\right)_{X(\mu)} = (\Delta E_{n+1,k}, \Delta p^1_{n+1,k}, \Delta p^2_{n+1,k}, \Delta p^3_{n+1,k}, 0)_{X(\mu)} \tag{63}$$

$$g_{X(\mu)}(\Delta\vec{p}^{(5)}_{X(\mu)}, \Delta\vec{p}^{(5)}_{X(\mu)}) = -(\Delta E_{n+1,k})^2 + (\Delta p^1_{n+1,k})^2 + (\Delta p^2_{n+1,k})^2 + (\Delta p^3_{n+1,k})^2 = 0 \tag{64}$$

□

Given Eq. (63) we here also have $\frac{d^2x^j}{d\mu^2} = 0$, so we associate the γ-ray with **constant** spatial momenta $\Delta p^\alpha_{n+1,k}$. The idea of the γ-ray implements a ray theory of light in this model, with the front of the ray being a photon. We thus conveniently ignore that phenomena like interference and diffraction require wave theory. But recall that the aim is to show that the EPT agrees with SR: in the framework of SR, photons are point-particles too!

Having modeled the *objects* in the universe of the EPT in terms of point-particles, time-like strings and gamma-rays, we are now ready to model the *elementary principles* of the EPT.

Interpretation 3.30 For integers $n \in \mathbb{Z}$ and $k \in S_\omega$, in the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ the expression

$$\models\ 0:\ {}^{EP}f_k^n \to\ {}^{NW}f_k^n \tag{65}$$

models the Elementary Principle of Nonlocal Equilibrium, the first of seven axioms of the EPT; here the symbol '0' refers to the function $0 : \mathcal{M} \to\ {}^*_+\mathbb{R}$, $0 : X \mapsto (0, \ldots, 0)$. Since ${}^{EP}f_k^n = s_k^n$, cf. Int. 3.25, this expression means that in the 5D IRF of the observer $\mathcal{O}$, the particle state of the k^{th} monad at the n^{th} degree of evolution, located at the spatiotemporal position $X_{n,k}$, transforms spontaneously into the time-like string ${}^{NW}f_k^n$, which over time occupies the open line segment $\overline{\Delta X}_{n,k}$. □

Interpretation 3.31 For integers $n \in \mathbb{Z}$ and $k \in S_\omega$, in the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ the expression

$$\models\ {}^{NW}f_k^n :\ {}^{EP}f_k^n \to\ {}^{NP}f_k^{n+1} \tag{66}$$

models the Elementary Principle of Nonlocal Mediation, the second of seven axioms of the EPT. Since we have ${}^{EP}f_k^n = s_k^n$, cf. Int. 3.25, this expression means that in the 5D IRF the observer $\mathcal{O}$, the time-like string ${}^{NW}f_k^n$ effects a transition from the particle state of the k^{th} monad at the n^{th} degree of evolution, located at the spatiotemporal position $X_{n,k}$ in the 5D IRF of the observer $\mathcal{O}$, to the point-particle ${}^{NP}f_k^{n+1}$ located at the spatiotemporal position $X_{n+1,k}$ in the 5D IRF of $\mathcal{O}$. This has to be taken that at $t = t_{n+1,k}$, the time-like string "collapses" into, i.e. transforms into, the point-particle ${}^{NP}f_k^{n+1}$. □

Interpretation 3.32 For integers $n \in \mathbb{Z}$ and $k \in S_\omega$, in the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ the expression

$$\models\ 0:\ {}^{NP}f_k^{n+1} \to \gamma_k^{n+1} \tag{67}$$

models the Elementary Principle of Local Equilibrium, the third of seven axioms of the EPT; here '0' has the same meaning as in Int. 3.30. This expression means that in 5D IRF of the observer $\mathcal{O}$, the point-particle ${}^{NP}f_k^{n+1}$ *spontaneously* emits a γ-ray γ_k^{n+1}. □

Interpretation 3.33 For integers $n \in \mathbb{Z}$ and $k \in S_\omega$, in the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ the expression

$$\models\ \gamma_k^{n+1} :\ {}^{NP}f_k^{n+1} \to\ s_k^{n+1} \tag{68}$$

models the Elementary Principle of Local Mediation, the fourth of seven axioms of the EPT. This expression means that in the 5D IRF of the observer $\mathcal{O}$, the emitted γ-ray γ_k^{n+1} causes the transition of the point-particle ${}^{NP}f_k^{n+1}$ to the particle state of the k^{th} monad at the $(n+1)^{\text{th}}$ degree of evolution. Note that $\operatorname{supp}{}^{NP}f_k^{n+1} = \operatorname{supp}{}^{EP}f_k^{n+1} = \{X_{n+1,k}\}$, cf. Ints. 3.23 and 3.28, so the discrete transition ${}^{NP}f_k^{n+1} \to\ {}^{EP}f_k^{n+1}$ involves no spatiotemporal displacement. The particle state of the k^{th} monad at the $(n+1)^{\text{th}}$ degree of evolution is then the starting point of the k^{th} process from the $(n+1)^{\text{th}}$ to the $(n+2)^{\text{th}}$ degree of evolution. □

At the level of abstractness of the EPT, the phase quanta in terms of which the elementary principles are stated are abstracted from their properties. In the present model, however, we have endowed the phase quanta with properties, in particular energy and spatial momentum. To exclude inapplicability to the physical world the formulation of conservation laws is required; this has the status of an additional postulate.

Postulate 3.34 (Conservation of 5-momentum) Recalling Agreement 3.2 on the use of indices, we start with the time-like string ${}^{NW}f_k^n$ with energy $E_{n,k}^{NW}$ and associated spatial momenta $p_{n,k}^\alpha$. Upon its collapse to the point-particle ${}^{NP}f_k^{n+1}$ the momenta are conserved, so we associate ${}^{NP}f_k^{n+1}$ with a 5-momentum

$$\vec{p}^{(5\uparrow)}_{X_{n+1,k}} := (E_{n+1,k}^{NP}, p_{n,k}^1, p_{n,k}^2, p_{n,k}^3, \chi_k \cdot m_k)_{X_{n+1,k}} \tag{69}$$

for which $g_{X_{n+1,k}}(\vec{p}^{(5\uparrow)}_{X_{n+1,k}}, \vec{p}^{(5\uparrow)}_{X_{n+1,k}}) = 0$, so that $E_{n+1,k}^{NP} = E_{n,k}^{NW} = \sqrt{\sum (p_{n,k}^\alpha)^2}$. The γ-ray γ_k^{n+1} with associated spatial momenta $\Delta p_{n+1,k}^\alpha$ emitted by the point-particle ${}^{NP}f_k^{n+1}$ then causes the latter to transform to the point-particle ${}^{EP}f_k^{n+1}$, so we associate ${}^{EP}f_k^{n+1}$ with a 5-momentum

$$\vec{p}^{(5\downarrow)}_{X_{n+1,k}} := (E_{n+1,k}^{EP}, p_{n,k}^1 - \Delta p_{n+1,k}^1, p_{n,k}^2 - \Delta p_{n+1,k}^2, p_{n,k}^3 - \Delta p_{n+1,k}^3, \chi_k \cdot m_k)_{X_{n+1,k}} \tag{70}$$

for which $g_{X_{n+1,k}}(\vec{p}^{(5\downarrow)}_{X_{n+1,k}}, \vec{p}^{(5\downarrow)}_{X_{n+1,k}}) = 0$ so that $E^{EP}_{n+1,k} = \sqrt{\sum(p^\alpha_{n,k} - \Delta p^\alpha_{n+1,k})^2}$. By a discrete state transition, the point-particle ${}^{EP}f^{n+1}_k$ subsequently transforms into the time-like string ${}^{NW}f^{n+1}_k$ with energy $E^{NW}_{n+1,k}$ and associated spatial momenta $p^\alpha_{n+1,k}$. If a γ-ray γ^p_m with associated spatial momenta $\Delta p^\alpha_{p,m}$ is absorbed, that is, if a γ-ray γ^p_m has a path $X(t) = (t, x(t), y(t), z(t), 0) \in \mathcal{M}$ such that

$$\lim_{t \to t_{n+1,k}} X(t) = X_{n+1,k} \tag{71}$$

then 5-momentum is conserved according to

$$p^\alpha_{n+1,k} = (\vec{p}^{(5\downarrow)}_{X_{n+1,k}})^\alpha + \Delta p^\alpha_{p,m} \tag{72}$$

If no γ-ray is absorbed, then Eq. (72) holds with $\Delta p^\alpha_{p,m} = 0$. $\square$

Definition 3.35 Let $G_{\mathbb{Z},\omega,\mathcal{O}} = \{{}^{EP}f^n_k,\ {}^{NW}f^n_k,\ {}^{NP}f^{n+1}_k, \gamma^{n+1}_k \mid n \in Z,\ k \in S_\omega\}$; then $\langle G_{\mathbb{Z},\omega,\mathcal{O}}\rangle$ is the **commutative monoid generated by the set $G_{\mathbb{Z},\omega,\mathcal{O}}$ under function addition**, for which

$$f + g : X \mapsto f(X) + g(X) \tag{73}$$

Note that $s^n_k \in \langle G_{\mathbb{Z},\omega,\mathcal{O}}\rangle$ since $s^n_k = {}^{EP}f^n_k$. $\square$

Remark 3.36 Formulas (65), (66), (67), and (68) describe *all* individual processes in the 5D IRF of the observer $\mathcal{O}$: there are no other processes (but see Rem. 3.42). In the EPT, the corresponding four elementary principles all use expressions of the form $\begin{bmatrix} a \\ \overline{a} \end{bmatrix} : \begin{bmatrix} x \\ \overline{x} \end{bmatrix} \begin{matrix} \rightarrow \\ \leftarrow \end{matrix} \begin{bmatrix} y \\ \overline{y} \end{bmatrix}$, which are notations for

$$\langle \begin{bmatrix} a \\ \overline{a} \end{bmatrix}, \begin{bmatrix} x \\ \overline{x} \end{bmatrix}, \begin{bmatrix} y \\ \overline{y} \end{bmatrix} \rangle \in R \tag{74}$$

where R is a ternary relation on a finitely generated communtative monoid $(\langle g_1, g_2, g_3, \ldots, g_\Omega\rangle, +)$; an individual $\begin{bmatrix} a \\ \overline{a} \end{bmatrix}$, $\begin{bmatrix} x \\ \overline{x} \end{bmatrix}$, or $\begin{bmatrix} y \\ \overline{y} \end{bmatrix}$ in an expression (74) can, thus, be a sum of generators g_j. In the present model $M_{\mathbb{Z},\omega,\mathcal{O}}$, however, by these formulas (65), (66), (67), and (68) this relation R is interpreted as a ternary relation $I_{\mathbb{Z},\omega,\mathcal{O}}(R)$ on the set $\langle G_{\mathbb{Z},\omega,\mathcal{O}}\rangle$. The relation R is mentioned in Def. 2.1. $\square$

Having described the elementary processes in this model, we can now interpret the unary existence relation M_E of the EPT, which is straightforward.

Interpretation 3.37 For any generator $f \in G_{\mathbb{Z},\omega,\mathcal{O}}$ and for any finite sum of generators $f_1 + \ldots + f_n \in \langle G_{\mathbb{Z},\omega,\mathcal{O}}\rangle$ the expressions

$$\models\ \mathbb{E}f \Leftrightarrow f \neq 0 \tag{75}$$

$$\models\ \mathbb{E}f_1 + \ldots + f_n \Leftrightarrow \mathbb{E}f_1 + \ldots + f_{n-1} \wedge ((\mathbb{E}f_n \wedge f_1 \neq f_n \wedge f_2 \neq f_n \wedge \ldots \wedge f_{n-1} \neq f_n) \vee f_n = 0) \tag{76}$$

model the existence relation for the objects in the 5D IRF of the observer $\mathcal{O}$, where '$\mathbb{E}f$' denotes $f \in \mathbb{E}$ with $\mathbb{E} = I_{\mathbb{Z},\omega,\mathcal{O}}(M_E)$. $\square$

So, in the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ we have $\mathbb{E}\ {}^{EP}f^n_k$ for any $n \in Z,\ k \in S_\omega$, but we do not necessarily have $\mathbb{E}\gamma^{n+1}_k$ for any $n \in Z,\ k \in S_\omega$. The point is that there may be elementary processes in which no γ-ray is emitted: in that case $\gamma^{n+1}_k = 0$, and thus $\neg\mathbb{E}\gamma^{n+1}_k$; formula (67) is then trivially true.

But let's go through the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution in the 5D IRF of the observer $\mathcal{O}$. Starting from $\mathbb{E}\ {}^{EP}f^n_k$, the principle (65) together with the conservation law (72) guarantees that

$$\mathbb{E}\ {}^{EP}f^n_k \Rightarrow \mathbb{E}\ {}^{NW}f^n_k \tag{77}$$

From there, the principle (66) together with the conservation law (69) guarantees that

$$\mathbb{E}\ {}^{NW}f^n_k \Rightarrow \mathbb{E}\ {}^{NP}f^{n+1}_k \tag{78}$$

The point-particle then possibly emits a γ-ray, but because $\Delta p^4_{n+1,k} = 0$ (cf. Int. 3.29) it is impossible that all energy is emitted. The principle (68) together with the conservation law (70) then guarantees that

$$\mathbb{E}\ {}^{NP}f^{n+1}_k \Rightarrow \mathbb{E}\ {}^{EP}f^{n+1}_k \tag{79}$$

From there, the k^{th} process from the $(n+1)^{\text{th}}$ to the $(n+2)^{\text{th}}$ degree of evolution starts and the above repeats.

Interpretation 3.38 For integers $n \in \mathbb{Z}$ and $k \in S_\omega$, the constant ψ_k^n of the EPT designates the *state of the* k^{th} *monad from the* n^{th} *to the* $(n+1)^{\text{th}}$ *degree of evolution*. In the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ we then have

$$\psi_k^n \xrightarrow{\mathcal{O}} t_k^n \tag{80}$$

$$t_k^n : \mathcal{M} \to {}^*_+\mathbb{R} \tag{81}$$

such that the expression

$$\models\ t_k^n = {}^{EP}f_k^n + {}^{NW}f_k^n \tag{82}$$

models the Elementary Principle of Binad Composition, the fifth of seven axioms of the EPT. Recall that in the EPT the constant $\beta_k^n \equiv {}^{EP}\Phi_k^n + {}^{NW}\Phi_k^n$ designates the *binad occurring in the* k^{th} *process from the* n^{th} *to the* $(n+1)^{\text{th}}$ *degree of evolution*; the expression (82), thus, means that the state of the binad β_k^n in the 5D IRF of the observer $\mathcal{O}$ is modeled by the monadic state t_k^n which is made up of the point-particle ${}^{EP}f_k^n$ and the time-like string ${}^{NW}f_k^n$. (In a more advanced model of the EPT the state of the binad $\beta_k^n = {}^{EP}\Phi_k^n + {}^{NW}\Phi_k^n$ may be identified with an aggregation of monadic states.) □

We are now finally in a position to reap the fruits of all the above definitions and interpretations by establishing contact between the language of this model of the EPT and existing physical language. The next two examples will formalize electrons and positrons in the present framework, but it works the same way for neutrons, antineutrons, protons, antiprotons, and all other massive particles and their antimatter counterparts.

Example 3.39 Suppose that the k^{th} monad, introduced in Def. 3.21, is an *electronic* monad: then the rest mass spectrum σ_k maps any degree of evolution n to the rest mass $\sigma_k(n) = m_k = m_e$ of an electron; the characteristic number of normality χ_k has then the value $+1$. The particle state s_k^n of the k^{th} monad at the n^{th} degree of evolution in the 5D IRF of the observer $\mathcal{O}$, introduced in Int. 3.23, is then an electron in a particle state: the lowest possible value of its energy $E_{n,k}^{EP}$ is the rest mass of an electron m_e, which is thus predetermined by the rest mass spectrum σ_k, and it is a *normal* particle state as indicated by the value $+1$ of the characteristic number of normality χ_k. This particle state of the electron is then the state of the phase quantum which marks the beginning of the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution, cf. Int. 3.25. In that process, on account of the principle stated in Int. 3.30, the particle state of the electron transforms by means of a discrete transition into the time-like string ${}^{NW}f_k^n$, which can be viewed as the time-like string state of the electron. In that state, the electron has at every point of the spatiotemporal extension of the time-like string state a momentum $p_{n,k}^4$ in the direction of the compact fifth dimension: as stated in Int. 3.27, for this momentum we have $p_{n,k}^4 = \chi_k \cdot m_e = m_e$, which is *positive* since $\chi_k = 1$ and $m_e > 0$. Together, the particle state of the electron and the string state of the electron form the state t_k^n, which is the state of the electron from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution—see Int. 3.38. As stated by Int. 3.31 the string state of the electron then "collapses" into the point-particle ${}^{NP}f_k^{n+1}$, which, after emission of a γ-ray γ_k^{n+1} as stated in Int. 3.32, transforms into the particle state s_k^{n+1} of that same electron the $(n+1)^{\text{th}}$ degree of evolution. That marks the beginning of the k^{th} process from the $(n+1)^{\text{th}}$ to the $(n+2)^{\text{th}}$ degree of evolution: the state t_k^{n+1} arising in that process is then the state of *that same electron* from the $(n+1)^{\text{th}}$ to the $(n+2)^{\text{th}}$ degree of evolution. Thus speaking, in this model the state of an electron *alternates* between a point-particle state and a time-like string state—as mentioned below Int. 3.27, the latter can be viewed as a wave state with the wave traveling in a straight line. □

Example 3.40 Suppose that the j^{th} monad is a *positronic* monad, then the rest mass spectrum σ_j is the same as that of an electronic monad: σ_j maps any degree of evolution n to the rest mass of an electron, so $\sigma_j(n) = m_j = m_e = \sigma_k(n)$. However, the characteristic number of normality χ_j has now the value -1. The particle state s_j^n of the j^{th} monad at the n^{th} degree of evolution in the 5D IRF of the observer $\mathcal{O}$ is then an positron in a particle state: the lowest possible value of its energy $E_{n,j}^{EP}$ is the rest mass of an electron m_e, which is thus predetermined by the rest mass spectrum σ_j, and it is an *abnormal* particle state as indicated by the value -1 of the characteristic number of normality χ_j. By the same mechanics as in Ex. 3.39, the state of a positron alternates between a point-particle state and a time-like string state. Contrary to the electron, however, the positron has in its time-like string state at every point of the spatiotemporal extension a *negative* momentum $p_{n,j}^4$ in the direction of the compact fifth dimension: for this momentum we have $p_{n,j}^4 = \chi_j \cdot m_j = -m_e$, which is *negative* since $\chi_j = -1$ and $m_e > 0$. In this as well as in the previous example, the characteristic number of normality has the same value as the lepton quantum number in quantum theory. □

By the same token free protons, free antiprotons, free neutrons, and free antineutrons can be formalized in the present model:

- for protons and antiprotons, the *protonic rest mass spectrum* predetermines the rest mass, and in each case the characteristic number of normality has the same value as the baryon quantum number in quantum theory; the state of a protonic/antiprotonic monad from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution is then the state of a proton/antiproton;
- for neutrons and free antineutrons, the *neutronic rest mass spectrum* predetermines the rest mass, and in each case the characteristic number of normality has the same value as the baryon quantum number in quantum theory; the state of a neutronic/antineutronic monad from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution is then the state of a neutron/antineutron.

By the same mechanism as described in Ex. 3.39, all alternate between a point-particle state and a time-like string state: any ordinary massive particle (electron, proton, etc.) is thus modeled as a form of 'normal' matter with a *positive* momentum in the direction of the compact fifth dimension, while any massive antiparticle (positron, antiproton, etc.) is modeled as a form of 'abnormal' matter with a *negative* momentum in the direction of the compact fifth dimension. This very feature will remain in any more elaborate model of the EPT that also incorporates interactions—if repulsive gravity exists at all, then in there lies its cause.

It remains to be established that the present model is a *deterministic* model of the EPT, which contains an elementary principle of choice. In the 5D IRF of the observer $\mathcal{O}$, a choice takes place at every event that a time-like string ${}^{NW}f_k^n$ with spatiotemporal extension $\overline{\Delta X}_{n,k}$ transforms into a point-particle ${}^{NP}f_k^{n+1}$ at position $X_{n+1,k}$. The time-like string corresponds to a displacement $\Delta X = (\Delta t_{n,k}, \Delta x_{n,k}, \Delta y_{n,k}, \Delta z_{n,k}, 0)$ in $\mathcal{M}$, but although we have $t_{n+1,k} = t_{n,k} + \Delta t_{n,k}$—see Eq. (56)—it **does not** follow from the foregoing that $X_{n+1,k} = X_{n,k} + \Delta X_{n,k}$. It is, thus, the principle of choice that guarantees continuity. That is to say: the point-particle ${}^{NP}f_k^{n+1}$ is *chosen* from a set of possibilities Θ_k^{n+1}.

Interpretation 3.41 Let Θ_k^{n+1} be the set of all functions ${}^{NP}h_k^{n+1} : \mathcal{M} \to {}^*_+\mathbb{R}$, whose support is a singleton $\{X\} \subset \mathcal{M}$ with $(X)^0 = (X_{n+1,k})^0 = t_{n+1,k}$ and $(X)^4 = (X_{n+1,k})^4 = 0$, and whose nonzero function value at X satisfies ${}^{NP}h_k^{n+1}(X) = {}^{NP}f_k^{n+1}(X)$. Let, for $P \in \mathcal{M}$ with $(P)^0 = (X_{n+1,k})^0$ and $(P)^4 = (X_{n+1,k})^4$, the choice function $\phi_P : \{\Theta_k^{n+1}\} \to \Theta_k^{n+1}$ be given by

$$\phi_P(\Theta_k^{n+1}) = {}^{NP}h_k^{n+1} \Leftrightarrow \text{supp } {}^{NP}h_k^{n+1} = \{P\} \tag{83}$$

Let $n \in \mathbb{Z}$, $k \in S_\omega$, and $X(t) \in \overline{\Delta X}_{n,k}$ with $t = (X)^0$; then in the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ the expression

$$\models {}^{NP}f_k^{n+1} = \phi_P(\Theta_k^{n+1}) \wedge P = \lim_{t \to t_{n+1,k}} X(t) = X_{n+1,k} \tag{84}$$

models the Elementary Principle of Choice, the sixth of seven axioms of the EPT. This expression means that in the 5D IRF of the observer $\mathcal{O}$, the point-particle ${}^{NP}f_k^{n+1}$ is a choice from a set of possibilities Θ_k^{n+1} strictly determined by the spatiotemporal extension $\overline{\Delta X}_{n,k}$ of the time-like string ${}^{NW}f_k^n$. See Fig. 2 for an illustration in a spacetime diagram. □

Remark 3.42 We leave constants ${}^{S}\Phi_k^{n+2}$, which designate the *spatial phase quanta* that occur in the universe of the EPT, **uninterpreted**; the same then goes for the Elementary Principle of Formation of Space, the last of seven axioms of the EPT. The reason for this omission is that these interpretations are not needed for showing that the EPT agrees with SR. For those who find this omission unacceptable, we can interpret a constant ${}^{S}\Phi_k^{n+2}$ as a function ${}^{S}f_k^{n+2} : \mathcal{M} \to {}^*_+\mathbb{R}$ for which ${}^{S}f_k^{n+2}(X) = \gamma_k^{n+1}(X - E_1)$ where $E_1 = (1,0,0,0,0) \in \mathcal{M}$. The Elementary Principle of Formation of Space, which involves a continuous process, then becomes the expression

$$\models \mathbb{E}\gamma_k^{n+1} \Rightarrow \mathbb{E}\ {}^{S}f_k^{n+2} \tag{85}$$

(with '$\mathbb{E}$' as in Int. 3.37, and with the assumption that the set $G_{\mathbb{Z},\omega,\mathcal{O}}$ now also contains the functions ${}^{S}f_k^{n+2}$) meaning that in the 5D IRF of the observer $\mathcal{O}$, an existing γ-ray leaves a (vanishing) trace of substantial space. To emphasize it: this is just to trivially complete the model. □

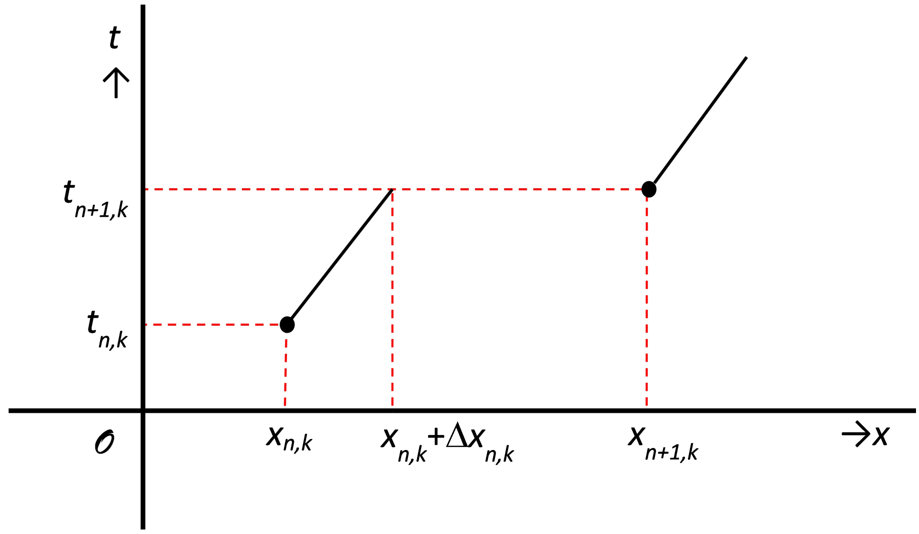

Figure 2: Spacetime diagram illustrating the elementary principle of choice, Eq. (84). Vertically the time-axis of the 5D IRF of observer $\mathcal{O}$, horizontally the x-axis; all other spatial dimensions are suppressed. The two black dots represent the positions $X_{n,k}$ and $X_{n+1,k}$ as indicated: these are the positions of the particle states s_k^n and s_k^{n+1} of the k^{th} monad at the n^{th} and the $(n+1)^{\text{th}}$ degree of evolution, respectively (cf. Int. 3.23). The two diagonal line segments represent the line segments $\overline{\Delta X}_{n,k}$ and $\overline{\Delta X}_{n+1,k}$ as indicated: these are the spatiotemporal extensions of the time-like strings ${}^{NW}f_k^n$ and ${}^{NW}f_k^{n+1}$, respectively (cf. Int. 3.27)—these time-like strings are created from these particle states by the transitions given by formula (65). The spacetime diagram shows a discontinuity: **without** the principle of choice there is no guarantee that $X_{n+1,k} = X_{n,k} + \Delta X_{n,k}$, so the transition from the time-like string ${}^{NW}f_k^n$ to the point-particle ${}^{NP}f_k^{n+1}$ at the position $X_{n+1,k}$ could then involve a discontinuity as shown in the diagram. But the principle of choice, as given by Int. 3.41, guarantees that $X_{n+1,k} = X_{n,k} + \Delta X_{n,k}$ and thus that **no such discontinuity occurs**. So in the 5D IRF of the observer $\mathcal{O}$, the particle state s_k^{n+1} is located where the spatiotemporal extension of the time-like string ${}^{NW}f_k^n$ ends. (In $M_{\mathbb{Z},\omega,\mathcal{O}}$, the higher black dot thus continues the lower line segment).

We can now conclude this section by defining the objects of the category $\mathscr{C}_{SR}$ along the lines of Def. 2.1:

Definition 3.43 An **object of the category** $\mathscr{C}_{SR}$ is a concrete set-theoretical model $M_{\mathbb{Z},\omega,\mathcal{O}}$ of the EPT, that is, a structure $\langle |M_{\mathbb{Z},\omega,\mathcal{O}}|, \mathbb{E}, I_{\mathbb{Z},\omega,\mathcal{O}}(R)\rangle$ consisting of:

(i) the set of individuals $|M_{\mathbb{Z},\omega,\mathcal{O}}|$, the *universe* of $M_{\mathbb{Z},\omega,\mathcal{O}}$, which is the union of the following sets:
- the set $A_{\omega,\mathcal{O}}$ specified by Def. 3.21;
- the set $\langle G_{\mathbb{Z},\omega,\mathcal{O}}\rangle$ specified by Ints. 3.23, 3.25, 3.27, 3.28, 3.29, Def. 3.35, and Rem. 3.42;
- the set $\Theta_{\mathbb{Z},\omega,\mathcal{O}} = \{\Theta_k^n \mid n \in Z, k \in S_\omega\}$ made up of the sets Θ_k^n specified by Int. 3.41;
- the set $\phi_{\mathbb{Z},\omega,\mathcal{O}}$ made up of the choice functions specified by Int. 3.41.

(ii) the unary existence relation $\mathbb{E}$ specified by Int. 3.37, which can be identified with a subset of $\langle G_{\mathbb{Z},\omega,\mathcal{O}}\rangle$;

(iii) the ternary relation $I_{\mathbb{Z},\omega,\mathcal{O}}(R)$ specified by Ints. 3.30, 3.31, 3.32, 3.33, and Rems. 3.36 and 3.42, which can be identified with a subset of $\langle G_{\mathbb{Z},\omega,\mathcal{O}}\rangle \times \langle G_{\mathbb{Z},\omega,\mathcal{O}}\rangle \times \langle G_{\mathbb{Z},\omega,\mathcal{O}}\rangle$.

In this structure, the axioms of the EPT are true. □

The collection of objects of $\mathscr{C}_{SR}$ is thus (uncountably) infinite; note that there is a class of objects for every value of ω. Which model applies to the physical world depends, then, on the system to be modeled: if an observer $\mathcal{O}$ wants to model a system that consists of just a single electron, which in the 5D IRF of $\mathcal{O}$ has a 5-momentum $\vec{p} = (m_e, 0, 0, 0, m_e)$ at position $X = (7, 2, 3, 4, \frac{1}{2})$, then a model applies with $\omega = 1$, such that $A_{1,\mathcal{O}}$ contains an electronic monad $\langle 1, \sigma_1, 1\rangle$, and such that $X = (7, 2, 3, 4, \frac{1}{2}) \in \text{supp}\ t_1^n$ with $\vec{p}_X = (m_e, 0, 0, 0, m_e)_X$ for some $t_1^n \in \langle G_{\mathbb{Z},1,\mathcal{O}}\rangle$.

3.4 The arrows of the category $\mathscr{C}_{SR}$

If for an inertial observer $\mathcal{O}$ a concrete set-theoretical model $M_{\mathbb{Z},\omega,\mathcal{O}}$ of the EPT applies to a given physical system, then for a *different* inertial observer a *different* model $M_{\mathbb{Z},\omega,\mathcal{O}'}$ applies *to the same physical system.* The point is, then, that these models are related by an arrow T in the collection of arrows of $\mathscr{C}_{SR}$. That being said, we can define precisely what such an arrow is along the lines of Def. 2.1.

Definition 3.44 Let the objects of the category $\mathscr{C}_{SR}$ be structures as in Def. 3.43. Then an **arrow of the category** $\mathscr{C}_{SR}$ is an isomorphism T of a structure $M_{\mathbb{Z},\omega,\mathcal{O}} = \langle |M_{\mathbb{Z},\omega,\mathcal{O}}|, \mathbb{E}, I_{\mathbb{Z},\omega,\mathcal{O}}(R)\rangle$ and a structure $M_{\mathbb{Z},\omega,\mathcal{O}'} = \langle |M_{\mathbb{Z},\omega,\mathcal{O}'}, \mathbb{E}, I_{\mathbb{Z},\omega,\mathcal{O}'}(R)\rangle$, which maps $|M_{\mathbb{Z},\omega,\mathcal{O}}|$ bijectively to $|M_{\mathbb{Z},\omega,\mathcal{O}'}|$ such that

$$T(f_1) + T(f_2) = T(f_1 + f_2) \quad \text{for any } f_1, f_2 \in \langle G_{\mathbb{Z},\omega,\mathcal{O}}\rangle \tag{86}$$

$$\mathbb{E}T(f) \Leftrightarrow \mathbb{E}f \quad \text{for any } f \in \langle G_{\mathbb{Z},\omega,\mathcal{O}}\rangle \tag{87}$$

$$\langle T(f_1), T(f_2), T(f_3)\rangle \in I_{\mathbb{Z},\omega,\mathcal{O}'}(R) \Leftrightarrow \langle f_1, f_2, f_3\rangle \in I_{\mathbb{Z},\omega,\mathcal{O}}(R) \quad \text{for any } f_1, f_2, f_3 \in \langle G_{\mathbb{Z},\omega,\mathcal{O}}\rangle \tag{88}$$

□

So, once we have a concrete set-theoretical model $M_{\mathbb{Z},\omega,\mathcal{O}}$ that applies to a given system for inertial observer $\mathcal{O}$, then the arrows of $\mathscr{C}_{SR}$ transform this to models $M_{\mathbb{Z},\omega,\mathcal{O}'}, M_{\mathbb{Z},\omega,\mathcal{O}''}, \ldots$ that will apply to *the same physical system* for other inertial observers $\mathcal{O}', \mathcal{O}'', \ldots$ That is, the arrows relate the predictions of observer $\mathcal{O}$ to those of observers $\mathcal{O}', \mathcal{O}'', \ldots$ This reproduces relativity of length and time as in standard SR.
There are, then, three kinds of special arrows ('ur-arrows') in the collection of arrows of $\mathscr{C}_{SR}$:

- **permutation arrows** that correspond to a permutation of counting numbers;
- **translation arrows** that correspond to a translation in 5D spacetime;
- **Lorentz arrows** that correspond to a 5D Lorentz transformation.

Below these ur-arrows will be defined precisely; all other arrows are then compositions of these ur-arrows. To define such an ur-arrow, it suffices to define how the individuals in the set $A_{\omega,\mathcal{O}}$ and the individuals in the set $G_{\mathbb{Z},\omega,\mathcal{O}}$ of generators of $\langle G_{\mathbb{Z},\omega,\mathcal{O}}\rangle$ transform: that determines everything else. To see that, let T be an arrow $T : M_{\mathbb{Z},\omega,\mathcal{O}} \to M_{\mathbb{Z},\omega,\mathcal{O}'}$; if $T({}^{NW}f^n_k)$ and $T({}^{NP}f^{n+1}_k)$ are known for all $n \in \mathbb{Z}, k \in S_\omega$, then $\Theta_{\mathbb{Z},\omega,\mathcal{O}'}$ and $\phi_{\mathbb{Z},\omega,\mathcal{O}'}$ are determined by Int. 3.41.

Definition 3.45 Let $M_{\mathbb{Z},\omega,\mathcal{O}}$ be a concrete set-theoretical model of the EPT, and let Σ_ω be the set of all permutations on the section of positive integers S_ω. Then for every $\pi \in \Sigma_\omega$ there is a **permutation arrow** $T_{\mathbb{Z},\omega,\mathcal{O},\pi}$ and a concrete set-theoretical model $M_{\mathbb{Z},\omega,\mathcal{O}'}$ of the EPT given by

$$T_{\mathbb{Z},\omega,\mathcal{O},\pi} : M_{\mathbb{Z},\omega,\mathcal{O}} \to M_{\mathbb{Z},\omega,\mathcal{O}'} \tag{89}$$

$$T_{\mathbb{Z},\omega,\mathcal{O},\pi} : \langle k, \sigma_k, \chi_k\rangle \mapsto \langle \pi(k), \sigma_{\pi(k)}, \chi_{\pi(k)}\rangle \wedge \sigma_{\pi(k)} = \sigma_k \wedge \chi_{\pi(k)} = \chi_k \tag{90}$$

$$T_{\mathbb{Z},\omega,\mathcal{O},\pi} :\ {}^{\alpha}f^n_k \mapsto {}^{\alpha}f'^{\,n}_{\pi(k)} \wedge {}^{\alpha}f^n_k = {}^{\alpha}f'^{\,n}_{\pi(k)} \tag{91}$$

(here α denotes EP, NP, NW, LW, S). □

Loosely speaking, for every inertial observer $\mathcal{O}$ there is an equivalent inertial observer $\mathcal{O}'$ such that the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution in the 5D IRF of $\mathcal{O}$ is the $\pi(k)^{\text{th}}$ process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution in the 5D IRF of $\mathcal{O}'$. The point is that the numerical value that an observer gives to the label k is trivial: it is only important that the same value is maintained for its successor and its predecessor, and for the events (i.e. the state transitions) in that process.

Definition 3.46 Let $M_{\mathbb{Z},\omega,\mathcal{O}}$ be a concrete set-theoretical model of the EPT. Then for every function τ for which $\tau : S_\omega \times \mathbb{Z} \to \mathbb{Z}$, $\tau : (k, n) \mapsto n + j(k)$, there is a **permutation arrow** $T_{\mathbb{Z},\omega,\mathcal{O},\tau}$ and a concrete set-theoretical model $M_{\mathbb{Z},\omega,\mathcal{O}'}$ of the EPT given by

$$T_{\mathbb{Z},\omega,\mathcal{O},\tau} : M_{\mathbb{Z},\omega,\mathcal{O}} \to M_{\mathbb{Z},\omega,\mathcal{O}'} \tag{92}$$

$$T_{\mathbb{Z},\omega,\mathcal{O},\tau} : \langle k, \sigma_k, \chi_k\rangle \mapsto \langle k, \sigma_k, \chi_k\rangle \tag{93}$$

$$T_{\mathbb{Z},\omega,\mathcal{O},\tau} :\ {}^{\alpha}f^n_k \mapsto {}^{\alpha}f'^{\,\tau(n,k)}_k \wedge {}^{\alpha}f^n_k = {}^{\alpha}f'^{\,n+j(k)}_k \tag{94}$$

(here α denotes EP, NP, NW, LW, S). □

Loosely speaking, for every inertial observer $\mathcal{O}$ there is an equivalent inertial observer $\mathcal{O}'$ such that the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution in the 5D IRF of $\mathcal{O}$ is the k^{th} process from the $(n+j(k))^{\text{th}}$ to the $(n+j(k)+1)^{\text{th}}$ degree of evolution in the 5D IRF of $\mathcal{O}'$. The point is that the numerical value that an observer gives to the degree of evolution n is trivial *in this categorical model*: only the displacement in degrees of evolution matters (vide infra).

Definition 3.47 Let $M_{\mathbb{Z},\omega,\mathcal{O}}$ be a concrete set-theoretical model of the EPT. Then for every $\Delta X \in \mathcal{M}$ with $(\Delta X)^4 = 0$ there is a **translation arrow** $T_{\mathbb{Z},\omega,\mathcal{O},\Delta X}$ and a concrete set-theoretical model $M_{\mathbb{Z},\omega,\mathcal{O}''}$ of the EPT given by

$$T_{\mathbb{Z},\omega,\mathcal{O},\kappa} : M_{\mathbb{Z},\omega,\mathcal{O}} \to M_{\mathbb{Z},\omega,\mathcal{O}''} \tag{95}$$

$$T_{\mathbb{Z},\omega,\mathcal{O},\kappa} : \langle k, \sigma_k, \chi_k\rangle \mapsto \langle k, \sigma_k, \chi_k\rangle \tag{96}$$

$$T_{\mathbb{Z},\omega,\mathcal{O},\kappa} : \ {}^{\alpha}f_k^n \wedge \ \mapsto \ {}^{\alpha}f''^{\,n}_k \wedge \ {}^{\alpha}f''^{\,n}_k(X) = \ {}^{\alpha}f_k^n(X + \Delta X) \tag{97}$$

(here α denotes EP, NP, NW, LW, S). □

Loosely speaking, for every inertial observer $\mathcal{O}$ there is an equivalent inertial observer $\mathcal{O}''$ who does not move relative to $\mathcal{O}$, such that the constituents of the 5D IRF of $\mathcal{O}''$ are the constituents of the 5D IRF of $\mathcal{O}$ shifted by ΔX. The set of monads $A_{\omega,\mathcal{O}}$ is thus invariant under translation.

Definition 3.48 Let $M_{\mathbb{Z},\omega,\mathcal{O}}$ be a concrete set-theoretical model of the EPT. Then for every operator Λ in the set $\mathcal{O}$ of operators from Def. 3.5 there is a **Lorentz arrow** $T_{\mathbb{Z},\omega,\mathcal{O},\Lambda}$ and a concrete set-theoretical model $M_{\mathbb{Z},\omega,\mathcal{O}'''}$ of the EPT given by

$$T_{\mathbb{Z},\omega,\mathcal{O},\Lambda} : M_{\mathbb{Z},\omega,\mathcal{O}} \to M_{\mathbb{Z},\omega,\mathcal{O}'''} \tag{98}$$

$$T_{\mathbb{Z},\omega,\mathcal{O},\Lambda} : \langle k, \sigma_k, \chi_k\rangle \mapsto \langle k, \sigma_k, \chi_k\rangle \tag{99}$$

$$T_{\mathbb{Z},\omega,\mathcal{O},\Lambda} : \ {}^{\alpha}f_k^n \mapsto \ {}^{\alpha}f'''^{\,n}_k \wedge \text{supp}\ {}^{\alpha}f'''^{\,n}_k = \Lambda[\text{supp}\ {}^{\alpha}f_k^n] \tag{100}$$

$$T_{\mathbb{Z},\omega,\mathcal{O},\Lambda} : \vec{p}_X \mapsto \Lambda^*(\vec{p})_{\Lambda(X)} \tag{101}$$

where α denotes EP, NP, NW, LW, S, $\vec{p}_X$ is any 5-momentum at the point X in the 5D IRF of the observer $\mathcal{O}$, and Λ^* denotes the linear operation on $\mathbb{R}^5$ with the same matrix as Λ. □

Loosely speaking, for every inertial observer $\mathcal{O}$ there is an equivalent inertial observer $\mathcal{O}'''$ who moves relative to $\mathcal{O}$ with constant speed, such that the origins of the 5D IRFs of $\mathcal{O}$ and $\mathcal{O}'''$ coincide, and such that the support of the individuals in $G_{\mathbb{Z},\omega,\mathcal{O}}$ and $G_{\mathbb{Z},\omega,\mathcal{O}'''}$, as well as the 5-momenta at any point in the support, are related by a 5D Lorentz transformation Λ. In other words, an object that has 5-momentum $\vec{p}$ at position X in the 5D IRF of $\mathcal{O}$ has 5-momentum $\Lambda^*(\vec{p})$ at position $\Lambda(X)$ in the 5D IRF of $\mathcal{O}'''$.

The collection of arrows of the categorical model is then generated by the ur-arrows defined above under arrow composition; for any arrows $T : \text{dom}\ T \to \text{cod}\ T$ and $T' : \text{dom}\ T' \to \text{cod}\ T'$ with $\text{cod}\ T' = \text{dom}\ T$ there is thus an arrow $T \circ T' : \text{dom}\ T' \to \text{cod}\ T$. See Fig. 3 for a diagrammatic illustration.

4 Discussion and conclusions

4.1 Proof that SR is incorporated in $\mathscr{C}_{SR}$

The daunting task that awaits us in this section is to prove that SR is incorporated in $\mathscr{C}_{SR}$. It is well known that SR derives from just two postulates—the universality of light speed, and the principle of relativity [19]—so our task here is to reformulate these two postulates in the language of $\mathscr{C}_{SR}$, and to prove that the resulting two expressions are theorems of $\mathscr{C}_{SR}$.

A first observation is then that the universe of discourse of SR concerns observers that model *the same physical reality*: such a universe of discourse is the first thing that we must define in $\mathscr{C}_{SR}$.

Definition 4.1 (Model class) Let $M_{\mathbb{Z},\omega,\mathcal{O}}$ and $M_{\mathbb{Z},\omega,\mathcal{O}'}$ be two set-theoretical models of the EPT that are objects of $\mathscr{C}_{SR}$. Then $M_{\mathbb{Z},\omega,\mathcal{O}}$ and $M_{\mathbb{Z},\omega,\mathcal{O}'}$ are equivalent, notation: $M_{\mathbb{Z},\omega,\mathcal{O}} \sim M_{\mathbb{Z},\omega,\mathcal{O}'}$, if and only if there is an isomorphism T between $M_{\mathbb{Z},\omega,\mathcal{O}}$ and $M_{\mathbb{Z},\omega,\mathcal{O}'}$:

$$M_{\mathbb{Z},\omega,\mathcal{O}} \sim M_{\mathbb{Z},\omega,\mathcal{O}'} \Leftrightarrow \exists T (T : M_{\mathbb{Z},\omega,\mathcal{O}} \to M_{\mathbb{Z},\omega,\mathcal{O}'}) \tag{102}$$

A **model class** $[M_{\mathbb{Z},\omega,\mathcal{O}}]_\sim$ is then the class of models that are isomorphic to $M_{\mathbb{Z},\omega,\mathcal{O}}$. □

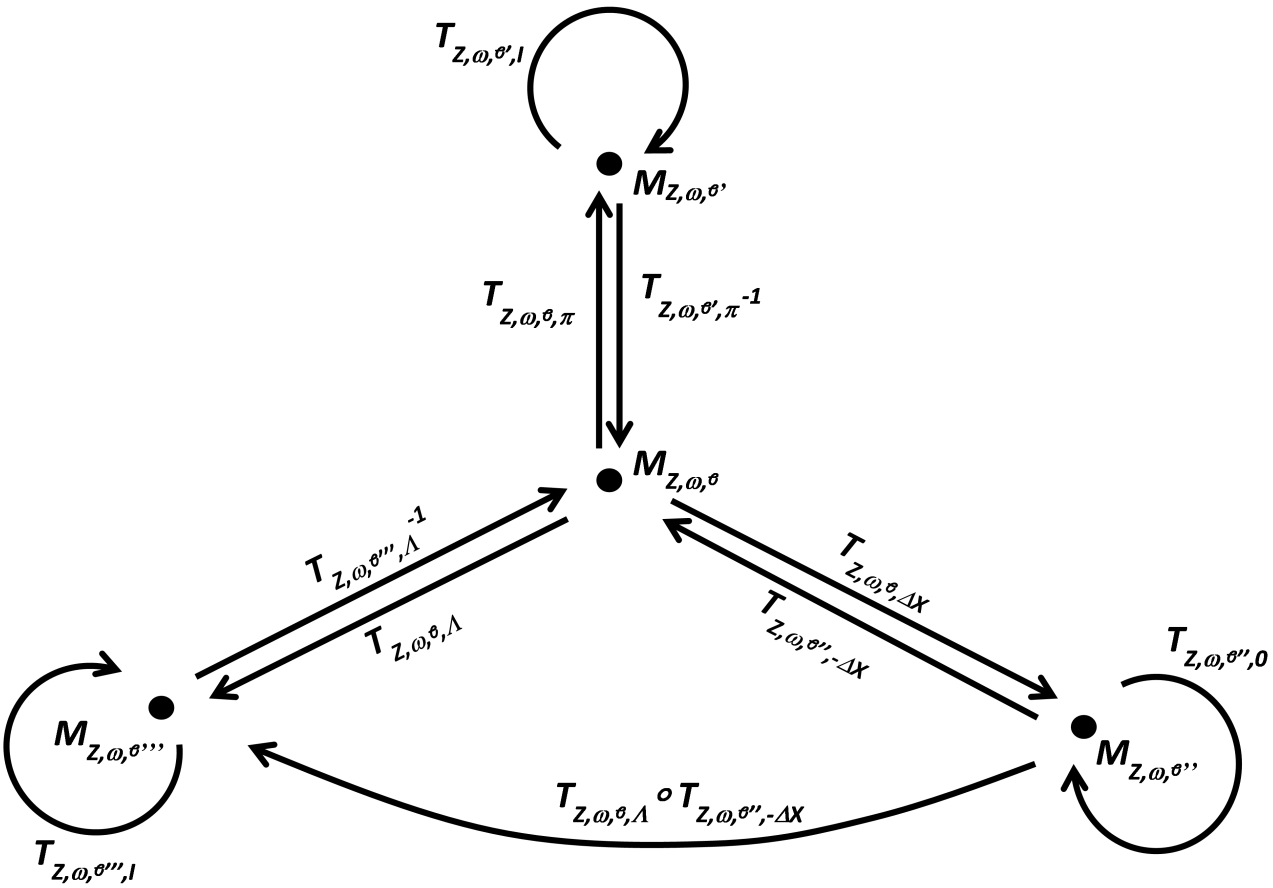

Figure 3: Diagram illustrating various ur-arrows, identity arrows and composite arrows in the categorical model. The four dots represent the models $M_{\mathbb{Z},\omega,\mathcal{O}}$, $M_{\mathbb{Z},\omega,\mathcal{O}'}$, $M_{\mathbb{Z},\omega,\mathcal{O}''}$, $M_{\mathbb{Z},\omega,\mathcal{O}'''}$ in the collection of objects as indicated. The vertical arrows between $M_{\mathbb{Z},\omega,\mathcal{O}}$ and $M_{\mathbb{Z},\omega,\mathcal{O}'}$ represent two permutation arrows $T_{\mathbb{Z},\omega,\mathcal{O},\pi} : M_{\mathbb{Z},\omega,\mathcal{O}} \to M_{\mathbb{Z},\omega,\mathcal{O}'}$ and $T_{\mathbb{Z},\omega,\mathcal{O}',\pi^{-1}} : M_{\mathbb{Z},\omega,\mathcal{O}'} \to M_{\mathbb{Z},\omega,\mathcal{O}}$ as defined by Def. 3.45. The circular arrow at the top middle is the identity arrow $T_{\mathbb{Z},\omega,\mathcal{O}',I} : M_{\mathbb{Z},\omega,\mathcal{O}'} \to M_{\mathbb{Z},\omega,\mathcal{O}'}$ corresponding with the identity permutation $I : S_\omega \to S_\omega$, $I : k \mapsto k$. Permutation arrows as defined by Def. 3.46 are not shown. The diagonal arrows between $M_{\mathbb{Z},\omega,\mathcal{O}}$ and $M_{\mathbb{Z},\omega,\mathcal{O}''}$ represent two translation arrows $T_{\mathbb{Z},\omega,\mathcal{O},\Delta X} : M_{\mathbb{Z},\omega,\mathcal{O}} \to M_{\mathbb{Z},\omega,\mathcal{O}''}$ and $T_{\mathbb{Z},\omega,\mathcal{O}'',-\Delta X} : M_{\mathbb{Z},\omega,\mathcal{O}''} \to M_{\mathbb{Z},\omega,\mathcal{O}}$ as defined by Def. 3.47. The circular arrow at the lower right is the identity arrow $T_{\mathbb{Z},\omega,\mathcal{O}'',0} : M_{\mathbb{Z},\omega,\mathcal{O}''} \to M_{\mathbb{Z},\omega,\mathcal{O}''}$ corresponding with the zero displacement in $\mathcal{M}$. The diagonal arrows between $M_{\mathbb{Z},\omega,\mathcal{O}}$ and $M_{\mathbb{Z},\omega,\mathcal{O}'''}$ represent two Lorentz arrows $T_{\mathbb{Z},\omega,\mathcal{O},\Lambda} : M_{\mathbb{Z},\omega,\mathcal{O}} \to M_{\mathbb{Z},\omega,\mathcal{O}'''}$ and $T_{\mathbb{Z},\omega,\mathcal{O}''',\Lambda^{-1}} : M_{\mathbb{Z},\omega,\mathcal{O}'''} \to M_{\mathbb{Z},\omega,\mathcal{O}}$ as defined by Def. 3.48. The circular arrow at the lower left is the identity arrow $T_{\mathbb{Z},\omega,\mathcal{O}''',I} : M_{\mathbb{Z},\omega,\mathcal{O}'''} \to M_{\mathbb{Z},\omega,\mathcal{O}'''}$ corresponding with the identity transformation $I : \mathcal{M} \to \mathcal{M}$, $I : X \mapsto X$. The bent arrow at the bottom represents the composite arrow $T_{\mathbb{Z},\omega,\mathcal{O},\Lambda} \circ T_{\mathbb{Z},\omega,\mathcal{O}'',-\Delta X} : M_{\mathbb{Z},\omega,\mathcal{O}''} \to M_{\mathbb{Z},\omega,\mathcal{O}'''}$; for the sake of clarity other (composite) arrows are omitted in the diagram. The diagram commutes.

Consider, for example, that an observer $\mathcal{O}$ models a system consisting of just one electron. Then there is a model $M_{\mathbb{Z},1,\mathcal{O}}$ where the electron moves unaccelerated in a given time interval, and there is at least one model $M'_{\mathbb{Z},1,\mathcal{O}}$ where the electron accelerates in that time interval. Which models applies depends on the actual physics, but the point is that $M_{\mathbb{Z},1,\mathcal{O}}$ and $M'_{\mathbb{Z},1,\mathcal{O}}$ are **not equivalent**: $[M_{\mathbb{Z},1,\mathcal{O}}]_\sim$ and $[M'_{\mathbb{Z},1,\mathcal{O}}]_\sim$ are thus *different* model classes. We mention without proof that a model class $[M_{\mathbb{Z},\omega,\mathcal{O}}]_\sim$ together with the corresponding arrows forms a *full subcategory* of $\mathscr{C}_{SR}$, of which all objects are both *initial* and *terminal*; see e.g. [20] for a precise definition of the italic terms.

A model class $[M_{\mathbb{Z},\omega,\mathcal{O}}]_\sim$ thus concerns observers that model <u>the same</u> physical system: Def. 4.1 thus provides us with an important tool that enables us to express the two postulates of SR in a universal background logic. Quantification over all observers then becomes quantification over all models in an arbitrary model class.

That being said, the next discussion item is the *logical form* of these expressions. As to the universality of the speed of light, the idea is then to express this as a formula of the form

(*) *for all* $M \in [M_{\mathbb{Z},\omega,\mathcal{O}}]_\sim : M \models \Psi$

where Ψ is then a formula expressing that the speed of light is 1 everywhere for the observer $\mathcal{O}$. To prove such an expression (*), it suffices to give a concrete example of a model $M_{\mathbb{Z},\omega,\mathcal{O}}$ in which Ψ holds, and to prove that if Ψ holds in that model, then it holds in an arbitrary other model $M_{\mathbb{Z},\omega,\mathcal{O}'}$ in $[M_{\mathbb{Z},\omega,\mathcal{O}}]_\sim$.

As to the principle of relativity, the idea is then to express this as a formula of the form

(**) *for all* $M \in [M_{\mathbb{Z},\omega,\mathcal{O}}]_\sim$ *there exists* $M' \in [M_{\mathbb{Z},\omega,\mathcal{O}}]_\sim : M \models \Upsilon$ *implies* $M' \models \Upsilon'$

where Υ and Υ' are then formulas that together express that no experiment can determine the absolute speed of an observer. The core of the principle of relativity is, namely, that time passes at a different rate for non-co-moving observers, so for any observer $\mathcal{O}$ for whom time passes at a rate $r_\mathcal{O}$ (to be expressed by Υ), there is an observer $\mathcal{O}'$ for whom time passes at a rate $r_{\mathcal{O}'}$ with $r_{\mathcal{O}'} \neq r_\mathcal{O}$ (to be expressed by Υ'). To prove such an expression (**), it suffices to assume that Υ holds in a model $M_{\mathbb{Z},\omega,\mathcal{O}}$, and to prove that this implies that there is then another model $M_{\mathbb{Z},\omega,\mathcal{O}'}$ in $[M_{\mathbb{Z},\omega,\mathcal{O}}]_\sim$ in which Υ' holds.

The final point is then the precise formulation of the expressions (*) and (**). To formulate (*), the only thing that we still need is the definition of a notion of 'light speed' in regular 3D space:

Definition 4.2 (3-speed of a γ-ray) Let $M_{\mathbb{Z},\omega,\mathcal{O}}$ be a set-theoretical model of the EPT that is an object of $\mathscr{C}_{SR}$, and let a symbol ∂_j for $j \in \{0,1,2,3,4\}$ denote $\frac{\partial}{\partial x^j}$. Then for any function $\gamma_k^{n+1} \in G_{\mathbb{Z},\omega,\mathcal{O}}$ for which $\mathbb{E}\gamma_k^{n+1}$ and for any $X \in \text{supp}\ \gamma_k^{n+1}$, the **3-speed of the γ-ray at X**, notation: $v_X^{(3)}$, is the real number

$$v_X^{(3)} := \sqrt{(\partial_0 x^1)^2 + (\partial_0 x^2)^2 + (\partial_0 x^3)^2} \in \mathbb{R} \tag{103}$$

□

We can now prove the principle of universality of light speed in $\mathscr{C}_{SR}$:

Theorem 4.3 (Principle of universality of light speed in $\mathscr{C}_{SR}$) Let $[M_{\mathbb{Z},\omega,\mathcal{O}}]_\sim$ be any class of equivalent models of the EPT. Then in any of those equivalent models, the 3-speed of any γ-ray at any point X of its path is 1. In a formula:

$$\forall M\ :\ M \models \forall n \in \mathbb{Z} \forall k \in S_\omega \forall X \in \mathcal{M}(X \in \text{supp}\ \gamma_k^{n+1} \Rightarrow v_X^{(3)} = 1) \tag{104}$$

where quantification is over $[M_{\mathbb{Z},\omega,\mathcal{O}}]_\sim$. □

Proof Let Int. 3.29 hold for the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ in $[M_{\mathbb{Z},\omega,\mathcal{O}}]_\sim$; let γ_k^{n+1} be an arbitrary γ-ray for which $\mathbb{E}\gamma_k^{n+1}$. Then from Eq. (63) we have $\Delta\vec{p}_X^{(5)} = \Delta E_{n+1,k} \cdot (\partial_0 x^0, \partial_0 x^1, \partial_0 x^2, \partial_0 x^3, 0)_X = (\Delta p^0_{n+1,k}, \ldots, \Delta p^4_{n+1,k})_X$ at any point $X \in \text{supp}\ \gamma_k^{n+1}$, so we have

$$v_X^{(3)} = \frac{1}{\Delta E_{n+1,k}} \sqrt{(\Delta p^1_{n+1,k})^2 + (\Delta p^2_{n+1,k})^2 + (\Delta p^3_{n+1,k})^2} \tag{105}$$

But Eq. (64) then yields that $v_X^{(3)} = 1$, so in the 5D IRF of the inertial observer $\mathcal{O}$, the 3-speed of light is 1 everywhere. To prove that this then also holds in the 5D IRF of any other inertial observer, it suffices to show that this then also holds in a model $M_{\mathbb{Z},\omega,\mathcal{O}'}$ in $[M_{\mathbb{Z},\omega,\mathcal{O}}]_\sim$ that is related to $M_{\mathbb{Z},\omega,\mathcal{O}}$ by a Lorentz arrow, cf. Def. 3.48; the proof is trivial for the other ur-arrows. Thus speaking, there is a 5D Lorentz transformation $\Lambda \neq I$ such that $T_{\mathbb{Z},\omega,\mathcal{O},\Lambda} : M_{\mathbb{Z},\omega,\mathcal{O}} \to M_{\mathbb{Z},\omega,\mathcal{O}'}$. But Λ leaves the 4^{th} spatial coordinate untouched, so the image of the path of any γ-ray in $M_{\mathbb{Z},\omega,\mathcal{O}}$, which lies in the 4D subspace $\mathcal{M}_0 = \{X \in \mathcal{M} \mid (X)^4 = 0\}$, lies also in $\mathcal{M}_0$. This corresponds with a usual Lorentz transformation in 4D Minkowski space, so the 3-speed of the γ-ray is invariant under the transformation. Ergo, in the 5D IRF of any such inertial observer $\mathcal{O}'$, the 3-speed of light is also 1 everywhere. □

To formulate (**), we need the definition of a notion of 'rate of time-passing' in regular 3D space, as well as the definition of equivalent phenomena:

Definition 4.4 (duration of a time-like string) Let $M_{\mathbb{Z},\omega,\mathcal{O}}$ be a set-theoretical model of the EPT that is an object of $\mathscr{C}_{SR}$. Then for any time-like string ${}^{NW}f_k^n \in G_{\mathbb{Z},\omega,\mathcal{O}}$ with supp ${}^{NW}f_k^n$ as in Eqs. (49)-(50), the **duration of the time-like string**, notation: $d({}^{NW}f_k^n)$, is the real number

$$d({}^{NW}f_k^n) = \Delta t_{n,k} \tag{106}$$

□

Definition 4.5 (equivalent phenomena) Let $[M_{\mathbb{Z},\omega,\mathcal{O}}]_\sim$ be any class of equivalent models of the EPT. Let $M, M' \in [M_{\mathbb{Z},\omega,\mathcal{O}}]_\sim$ and let f be any function $f \in |M|$ for which $\mathbb{E}f$. Then **the equivalent of f in** M', notation: $f^\sim_{M'}$, is the image of f under the isomorphism $T : M \to M'$. (Note that there is only one such isomorphism). □

It has to be taken that equivalent phenomena are states of the *same* phase quantum in the 5D IRFs of *different* observers. We can now prove the principle of relativity in $\mathscr{C}_{SR}$:

Theorem 4.6 (Principle of relativity in $\mathscr{C}_{SR}$) Let $[M_{\mathbb{Z},\omega,\mathcal{O}}]_\sim$ be any class of equivalent models of the EPT. Then for any model M in which the time-like string ${}^{NW}f_k^n$ has duration $d({}^{NW}f_k^n)$, there is a model M' in which the duration of the equivalent of ${}^{NW}f'^{\,n}_k$ is not the same as $\Delta t_{n,k}$. In a formula:

$$\forall M \exists M' \ : \big(\ M \models d({}^{NW}f_k^n) = \Delta t_{n,k}\big) \Rightarrow \big(M' \models d(({}^{NW}f_k^n)^\sim_{M'}) \neq \Delta t_{n,k}\big) \tag{107}$$

where quantification is over $[M_{\mathbb{Z},\omega,\mathcal{O}}]_\sim$ and n, k are constants $n \in \mathbb{Z}, k \in S_\omega$. □

Proof Let M be any model $M \in [M_{\mathbb{Z},\omega,\mathcal{O}}]_\sim$, and let $M \models d({}^{NW}f_k^n) = \Delta t_{n,k}$; note that $\Delta t_{n,k} \geq 1$. Let the model M be associated with the 5D IRF of an observer $\mathcal{O}$.

If $\Delta t_{n,k} > 1$, then there is another observer $\mathcal{O}'$ such that the 5D IRFs of $\mathcal{O}$ and $\mathcal{O}'$ are related by a 5D Lorentz transformation Λ, and such that $\mathcal{O}'$ has constant 3-speed $v^{(3)} = \sqrt{(\frac{\Delta x_{n,k}}{\Delta t_{n,k}})^2 + (\frac{\Delta y_{n,k}}{\Delta t_{n,k}})^2 + (\frac{\Delta z_{n,k}}{\Delta t_{n,k}})^2}$ in the 5D IRF of $\mathcal{O}$. Then there is a $M' \in [M_{\mathbb{Z},\omega,\mathcal{O}}]_\sim$ and a Lorentz arrow $T : M \to M'$ such that T corresponds with the 5D Lorentz transformation Λ. We then have $M' \models d(({}^{NW}f_k^n)^\sim_{M'}) = 1 \neq \Delta t_{n,k}$ as requested.

If $\Delta t_{n,k} = 1$, then there is another observer $\mathcal{O}'$ such that the 5D IRFs of $\mathcal{O}$ and $\mathcal{O}'$ are related by a 5D Lorentz transformation Λ, and such that $\mathcal{O}'$ has constant 3-speed $v^{(3)} > 0$ in the 5D IRF of $\mathcal{O}$. Then there is a $M' \in [M_{\mathbb{Z},\omega,\mathcal{O}}]_\sim$ and a Lorentz arrow $T : M \to M'$ such that T corresponds with the 5D Lorentz transformation Λ. We then have $M' \models d(({}^{NW}f_k^n)^\sim_{M'}) > 1 = \Delta t_{n,k}$ as requested. □

It should be realized that Th. 4.6 implies that no experiment can determine the absolute 3-speed of an observer. So, with Ths. 4.3 and 4.6 the postulates of standard SR have been expressed in the language of $\mathscr{C}_{SR}$ and proven.

With the above, the task that we had set out for ourselves at the beginning of this section has been carried out. However, we want to establish a firmer contact with the world view of standard SR by also formalizing notions of 'events', 'massive particles', and 'massless particles' in the language of $\mathscr{C}_{SR}$. We then state some lemmas that gives some understanding of the present result in terms of events and particles.

Definition 4.7 (Events) In the 5D IRF of an inertial observer $\mathcal{O}$, an **event** $\mathcal{E}$ is the manifestation of a discrete transition $g_1 \to g_2$ at a spatiotemporal position X in the 5D IRF of $\mathcal{O}$; we formalize an event $\mathcal{E}$ as a three-tuple $\langle \alpha^1, \alpha^2, \alpha^3 \rangle$ for which

$$\mathcal{E} = \langle X, I_{\mathbb{Z},\omega,\mathcal{O}}(g_1), I_{\mathbb{Z},\omega,\mathcal{O}}(g_2) \rangle \tag{108}$$

An event $\mathcal{E}$ in the 5D IRF of an inertial observer $\mathcal{O}$ and an event $\mathcal{E}'$ in the 5D IRF of an equivalent inertial observer $\mathcal{O}'$ are **equivalent**, notation: $\mathcal{E} \sim \mathcal{E}'$, if and only if $\mathcal{E}$ and $\mathcal{E}'$ are manifestations of *the same* discrete transition in the 5D IRFs of $\mathcal{O}$ adn $\mathcal{O}'$, respectively. □

Def. 4.7 provides a connection to the language of SR: both in the present context and in the context of standard SR we now can speak of *an event at a certain position.*

Notation 4.8 An expression '$\mathcal{E} \xrightarrow{\mathcal{O}} X$', meaning: 'for the observer $\mathcal{O}$ the event $\mathcal{E}$ takes place at spatiotemporal position X', is a notation for '$M_{\mathbb{Z},\omega,\mathcal{O}} \models (\mathcal{E})^1 = X$', that is, the first component of the three-tuple $\mathcal{E}$ is X. (This notation is based on a notation used in [19].) □

Thus speaking, in the 5D IRF of an observer $\mathcal{O}$, we have:

- discrete transitions ${}^{EP}f_k^n \to {}^{NW}f_k^n$ at the positions $X_{n,k}$: these are, thus, events $\mathcal{E}_{{}^{EP}f_k^n \to {}^{NW}f_k^n}$ for which $\mathcal{E}_{{}^{EP}f_k^n \to {}^{NW}f_k^n} \xrightarrow{\mathcal{O}} X_{n,k}$;
- discrete transitions ${}^{NW}f_k^n \to {}^{NP}f_k^{n+1}$ at the positions $X_{n+1,k}$: these are, thus, events $\mathcal{E}_{{}^{NW}f_k^n \to {}^{NP}f_k^{n+1}}$ for which $\mathcal{E}_{{}^{NW}f_k^n \to {}^{NP}f_k^{n+1}} \xrightarrow{\mathcal{O}} X_{n+1,k}$;
- discrete transitions ${}^{NP}f_k^{n+1} \to \gamma_k^{n+1}$ at positions $X_{n+1,k}$: these are, thus, events $\mathcal{E}_{{}^{NP}f_k^{n+1} \to \gamma_k^{n+1}}$ for which $\mathcal{E}_{{}^{NP}f_k^{n+1} \to \gamma_k^{n+1}} \xrightarrow{\mathcal{O}} X_{n+1,k}$;
- discrete transitions ${}^{NP}f_k^{n+1} \to {}^{EP}f_k^{n+1}$ at positions $X_{n+1,k}$: these are, thus, events $\mathcal{E}_{{}^{NP}f_k^{n+1} \to {}^{EP}f_k^{n+1}}$ for which $\mathcal{E}_{{}^{NP}f_k^{n+1} \to {}^{EP}f_k^{n+1}} \xrightarrow{\mathcal{O}} X_{n+1,k}$.

The point here is that in particular the absorption and emission of a γ-ray is an event: if γ-rays are absorbed, it is at these events $\mathcal{E}_{{}^{EP}f_k^n \to {}^{NW}f_k^n}$; if γ-rays are emitted, it is at these events $\mathcal{E}_{{}^{NP}f_k^{n+1} \to \gamma_k^{n+1}}$.

Definition 4.9 (Massive particles) Let $M_{\mathbb{Z},\omega,\mathcal{O}}$ be a set-theoretical model of the EPT that is an object of $\mathscr{C}_{SR}$. Then for any $k \in S_\omega$, the function t_k, for which

$$t_k : \mathcal{M} \to_+^* \mathbb{R}\ ,\ t_k = \sum_{n=-\infty}^{\infty} t_k^n \tag{109}$$

represents the k^{th} **massive particle** in the 5D IRF of $\mathcal{O}$, moving on a **5D world line** ℓ_k for which

$$\ell_k = \text{supp } t_k = \bigcup\{\{X_{n,k}\}, \overline{\Delta X}_{n,k} \mid n \in \mathbb{Z}\} \tag{110}$$

At any $X \in \ell_k$ where ℓ_k is differentiable, the **normal 5-velocity** $\vec{v}_X^{(5)}$ and **characteristic 5-velocity** $\vec{u}_X^{(5)}$ are given by

$$\vec{v}_X^{(5)} = (\partial_0 x^0, \ldots, \partial_0 x^4)_X = (1, v^1, v^2, v^3, v^4)_X \in T_X(\mathcal{M}) \tag{111}$$

$$\vec{u}_X^{(5)} = \frac{1}{m_k} \cdot (\vec{p})_X = (u^0, u^1, u^2, u^3, \chi_k)_X \in T_X(\mathcal{M}) \tag{112}$$

where the mass m_k and the characteristic number of normality χ_k are given by Def. 3.21; for the characteristic 5-velocity $\vec{u}_X^{(5)}$ we thus always have that $u^4 = \chi_k$. □

Note that $t_k^n = {}^{EP}f_k^n + {}^{NW}f_k^n$, so it is true that we have treated a time-like string ${}^{NW}f_k^n$ as one individual in the language of $M_{\mathbb{Z},\omega,\mathcal{O}}$, but the idea is thus that all the sequential time-like strings ${}^{NW}f_k^n$ and point-particles ${}^{EP}f_k^n$ add up to something that we can view as a massive particle moving on a 5D world line. But we keep in mind that the massive particles alternate between a particle state and a wave state.

Definition 4.10 (Massless particles) Let $M_{\mathbb{Z},\omega,\mathcal{O}}$ be a set-theoretical model of the EPT that is an object of $\mathscr{C}_{SR}$. Then any function $\gamma_k^{n+1} \in G_{\mathbb{Z},\omega,\mathcal{O}}$ for which $\mathbb{E}\gamma_k^{n+1}$ represents a **massless particle** in the 5D IRF of the inertial observer $\mathcal{O}$, moving on a **5D world line** ℓ for which

$$\ell = \text{supp } \gamma_k^{n+1} = \ell_{n+1,k}^{\gamma} \tag{113}$$

At any point $X = (x^0, \ldots, x^4) \in \ell_{n+1,k}^{\gamma}$ where $\ell_{n+1,k}^{\gamma}$ is differentiable, its **normal 5-velocity** $\vec{v}_X^{(5)}$ is given by

$$\vec{v}_X^{(5)} = (\partial_0 x^0, \ldots, \partial_0 x^4)_X = (1, v^1, v^2, v^3, 0)_X \in T_X(\mathcal{M}) \tag{114}$$

which is identical to its **characteristic 5-velocity** (so $\vec{u}_X^{(5)} = \vec{v}_X^{(5)}$ for a massless particle). □

Thus speaking, a γ-ray can be viewed as a massless particle moving on a 5D world line.

As promised, we will now state some lemmas that give an understanding of the categorical model $\mathscr{C}_{SR}$ in terms of particles and events. These lemmas will be states without proof, and a rigorous formulation, such as Ths. 4.3 and 4.6, will be omitted. From the perspective of the semantic view on theories, the objects of $\mathscr{C}_{SR}$ correspond to a theory—*in casu* a 5D account of SR. The lemmas below can thus also be viewed as lemmas of that 5D account of SR.

Lemma 4.11 (Motion on 5D null paths) For any inertial observer $\mathcal{O}$, any particle—massive or massless—moves on a continuous, piecewise differentiable 5D null path in the 5D IRF of $\mathcal{O}$, so that we have

$$\eta^{(5)}(\vec{v}_X^{(5)}, \vec{v}_X^{(5)}) = 0 \tag{115}$$

for the normal five-velocity $\vec{v}_X^{(5)}$ at any spatiotemporal position X on any particle's 5D world line ℓ in the 5D IRF of $\mathcal{O}$ (provided ℓ is differentiable at X).(See [21] for a definition of a continuous piecewise differentiable function.) □

Remark 4.12 In the physics literature on 5D theories it has become custom to refer to particles traveling on 5D null paths as 'massless'. Here the position is taken that this is an improper recontextualisation of the term 'massless'. It is true that in the context of 4D theories, such as standard SR, all particles that travel on null paths are massless. However, that doesn't make it true that all particles that travel on 5D null paths are massless too. The point is that 'mass' is a property *of the particle*, not of *its world line*. In the present context massive particles travel on a 5D null path, yet they do have the property 'mass': this is the (absolute value of the) momentum in the direction of the compact fifth dimension. It would be plain wrong to call these objects 'massless'. □

Lemma 4.13 (Universality of particle speed in $\mathscr{C}_{SR}$) For any inertial observer $\mathcal{O}$, any particle—massive or massless—moves with the speed of light through 4D space. That is, at any point X on its world line ℓ where its normal 5-velocity $\vec{v}_X^{(5)}(1, v^1, v^2, v^3, v^4)_X$ is defined, we have

$$(v^1)^2 + (v^2)^2 + (v^3)^2 + (v^4)^2 = 1 \tag{116}$$

□

Lemma 4.14 (Piecewise unaccelerated motion in $\mathscr{C}_{SR}$) For any inertial observer $\mathcal{O}$, any particle—massive or massless—moves piecewise unaccelerated; that is, at any point X of any particle's 5D world line ℓ we have for the instantaneous 5-acceleration

$$\vec{a}_X^{(5)} = (\partial_0\partial_0 x^0, \ldots, \partial_0\partial_0 x^4)_X = (0, 0, 0, 0, 0)_X \tag{117}$$

provided ℓ is differentiable at X. □

Remark 4.15 One should realize, however, that the fact that the motion of massive particles is piecewise unaccelerated as defined in Lemma 4.14 **does not imply** that there is no accelerated motion. It is merely the case that if we want to speak about a '5-acceleration' in the present context, then this has to be understood in terms of a change in, say, the characteristic 5-velocity of a particle on subsequent pieces of its 5D world line. E.g. at a point $X_{n+1,k}$ on the 5D world line ℓ_k of the k^{th} massive particle in the 5D IRF of an inertial observer $\mathcal{O}$, we can define the components $(\vec{a}_{n+1,k})^j$ of a 5-acceleration $\vec{a}_{n+1,k}$ as $(\vec{a}_{n+1,k})^j = (\vec{u}_{X'}^{(5)})^j - (\vec{u}_X^{(5)})^j$ where X and X' are positions on the line segments $\overline{\Delta X}_{n,k}$ and $\overline{\Delta X}_{n+1,k}$, respectively. (Note that a substraction $\vec{u}_{X'}^{(5)} - \vec{u}_X^{(5)}$ is undefined since these vectors are elements of *different* tangent spaces!) But this is just a suggestion on which we will not elaborate as it is not needed for the purpose of this paper. □

Lemma 4.16 (Invariant displacement in $\mathscr{C}_{SR}$) For any inertial observer $\mathcal{O}$, for any particle, massive or massless, the total distance traveled in the compact fifth dimension between any two events $\mathcal{E}_1$ and $\mathcal{E}_2$ on its 5D world line is observer-invariant. That is, for any inertial observer $\mathcal{O}$, for any two events $\mathcal{E}_1 \xrightarrow{\mathcal{O}} (t_1, x_1, y_1, z_1, n_1)$ and $\mathcal{E}_2 \xrightarrow{\mathcal{O}} (t_2, x_2, y_2, z_2, n_2)$ on any particle's 5D world line ℓ with $t_2 > t_1$, and for any inertial observer $\mathcal{O}'$ and the *equivalent* events $\mathcal{E}_1' \xrightarrow{\mathcal{O}'} (t_1', x_1', y_1', z_1', n_1')$, $\mathcal{E}_1' \sim \mathcal{E}_1$, and $\mathcal{E}_2' \xrightarrow{\mathcal{O}'} (t_2', x_2', y_2', z_2', n_2')$, $\mathcal{E}_2' \sim \mathcal{E}_2$, we have

$$\Delta n = \int_{t_1}^{t_2} |v^4| dt = \int_{t_1'}^{t_2'} |v^{4\prime}| dt \tag{118}$$

where v^4, v'^4 are coordinates of the normal 5-velocities $\vec{v}_X^{(5)} = (1, v^1, v^2, v^3, v^4)_X$ and $\vec{v}'^{(5)}_{X'} = (1, v^{1\prime}, v^{2\prime}, v^{3\prime}, v^{4\prime})_{X'}$ of the particle on positions X and X' on the world lines ℓ and ℓ' in the 5D IRFs of $\mathcal{O}$ and $\mathcal{O}'$. □

Lemma 4.17 (Correspondence to 4D SR) Under the projection $\pi : \mathcal{M} \to \mathbb{R}^4$, $\pi : (t, x, y, z, n) \mapsto (t, x, y, z)$ the 5D world lines of particles in the 5D IRF of an observer $\mathcal{O}$ yield the world lines of the particles in the 'usual' 4D Minkowski space with a metric g with signature $(-, +, +, +)$. □

Lemma 4.18 (Interpretation of the 'invariant interval' in $\mathscr{C}_{SR}$) Let π be the projection of Lemma 4.17, and let, for any observer $\mathcal{O}$, the events $\mathcal{E}_1 \xrightarrow{\mathcal{O}} (t_1, x_1, y_1, z_1, n_1)$ and $\mathcal{E}_2 \xrightarrow{\mathcal{O}} (t_2, x_2, y_2, z_2, n_2)$ be any two events on the 5D world line of any particle in the 5D IRF of $\mathcal{O}$, with $t_2 > t_1$; then the invariant interval Δs of the corresponding 4D spatiotemporal displacement vector $\Delta\vec{x} = \pi((t_2, x_2, y_2, z_2, n_2)) - \pi((t_1, x_1, y_1, z_1, n_1))$, for which

$$\Delta s = \sqrt{-g(\Delta\vec{x}, \Delta\vec{x})} \tag{119}$$

is identical the total distance Δn that the particle has traveled in the compact fifth dimension in the 5D IRF of $\mathcal{O}$ as given by Eq. (118). □

Lemma 4.19 (Philosophy of Time in $\mathscr{C}_{SR}$) For any observer $\mathcal{O}$ and for any two events $\mathcal{E}_1 \xrightarrow{\mathcal{O}} (t_1, x_1, y_1, z_1, n_1)$ and $\mathcal{E}_2 \xrightarrow{\mathcal{O}} (t_2, x_2, y_2, z_2, n_2)$ on the 5D world line of any particle in the 5D IRF of $\mathcal{O}$, with $t_2 > t_1$, the duration of the particles displacement $\Delta t = t_2 - t_1$ is nothing but the Euclidean measure of the displacement in 3D space and the total distance Δn that the particle has traveled in the compact fifth dimension in the 5D IRF of $\mathcal{O}$:

$$\Delta t = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2 + \Delta n^2} \tag{120}$$

where $\Delta x = x_2 - x_1$, $\Delta y = y_2 - y_1$, and $\Delta z = z_2 - z_1$. □

Concluding, this section has proven that SR is incorporated in the 5D categorical model $\mathscr{C}_{SR}$ of the EPT.

4.2 Relation to other 5D theories

The objective of this section is to identify core differences with other 5D theories. So, we're not out to prove that, for example, Lemma 4.14 doesn't hold in the framework of this or that other 5D theory: we will not look at the details, but at the ideas of the dimensions.

4.2.1 Kaluza-Klein theory

In Kaluza-Klein theory, published in the two papers [22, 23], the fifth dimension is also compact. So, beforehand we cannot exclude that the set $\mathcal{M}$ of Def. 3.1 applies as the set of all spatiotemporal positions in the "Kaluza-Klein universe" in the limit case that interactions can be neglected. However, in Kaluza-Klein theory the momentum p^4 of a massive particle in the direction of the compact fifth dimension is proportional to its electric charge q:

$$p^4 \propto q \tag{121}$$

In the present framework, however, the momentum p^4 of an ordinary (i.e., with characteristic number of normality $\chi_k = 1$) massive particle in the direction of the compact fifth dimension is *always* identical to its rest mass, cf. Int. 3.27 and Def. 4.9:

$$p^4 = m_0 \tag{122}$$

That means that in the present framework electrons and protons propagate *in the same direction* through the compact fifth dimension, while in the framework of Kaluza-Klein theory protons and electrons propagate *in opposite directions* through the compact fifth dimension. So even though in both cases the same mathematical set of coordinates may be applied for the compact fifth dimension, the inevitable conclusion is that the physical idea of the fifth dimension in Kaluza-Klein theory differs *fundamentally* from that of in the present framework of the EPT.

In 2000, Jennings has proposed to modify Kaluza-Klein theory so that "all physical propagation occurs at the speed of light" [24]. So then, the present Lemma 4.13 would hold in the framework of a modified Kaluza-Klein theory. However, no follow-up on this proposal has been published; given Eqs. (116) and (121), it is not clear what the momentum of a neutron in the direction of the compact fifth dimension would have to be in the modified Kaluza-Klein theory—Jennings' proposal seems irreconcilable with the existence of neutrons.

4.2.2 Wesson's Space-Time-Mass theory

In Wesson's Space-Time-Mass theory, published in [25], we also have that particles move on 5D null paths: Lemma 4.13 and Eq. 122 hold also in the framework of Wesson's theory. However, in Wesson's theory the fifth dimension is neither compact nor spatial; the fourth coordinate x^4 is for any particle proportional to its rest mass:

$$x^4 \propto m_0 \tag{123}$$

That means that particles *only* propagate through the fifth dimension when their rest mass changes. In the present framework, however, it is not only the case that the fifth dimension is both compact and spatial, but it is also the case that a massive particle propagates through the fifth dimension, as in Eq. (122), *even though* its rest mass doesn't change. The inevitable conclusion is that the physical idea of the fifth dimension in Wesson's theory differs *fundamentally* from that of in the present framework of the EPT.

4.2.3 5D unification scheme by Capozziello et al.

In the 5D unification scheme by Capozziello, Basini, and De Laurentis, published in [26], we also have five dimensions but these are five *equivalent* dimensions. The essence of the approach is that 'our' 4D world, in which we are able to distinguish among space, time, and mass, is obtained from a symmetry breaking, which is a sort of violation of Noether's theorem. World lines of protons and electrons in 4D spacetime are then images of projections of 5D null paths existing in the 5D space.

Since the five equivalent dimensions are not compact, this approach is very different from the present framework in which the five dimensions are not equivalent and one is compact.

4.2.4 Bordé's 5D theory

In Bordé's 5D theory, published in [27], we again have that all particles move on 5D null paths, so Eq. (116) holds. However, in the framework of Bordé's 5D theory, motion of a massive particle in the fifth dimension corresponds to a phase shift in its quantum mechanical wave function. The idea is that a wave function ψ has an extra phase factor that depends on proper time τ, as in

$$\psi(x) = \phi(x) \cdot \exp\left[\frac{im_0}{\hbar}(\tau - \tau_0)\right] \tag{124}$$

Bordé postulates that an infinitesimal displacement in the fifth dimension is *numerically* identical to an infinitesimal displacement in proper time:

$$ds = d\tau \tag{125}$$

The momentum in the fifth dimension can then be calculated with the momentum operator, which e.g. for the wave function (124) gives

$$\frac{\hbar\partial}{i\partial s}\psi = m_0 \cdot \psi \tag{126}$$

so rest mass m_0 is the momentum of a massive particle in the direction of the fifth dimension, as in Eq. (122).

But despite the fact that Eqs. (116) and (122) hold in the framework of Bordé's 5D theory, the physical idea of the fifth dimension is still fundamentally different from that of the present framework of the EPT. A fundamental difference is that in Bordé's 5D theory, a massive particle has a momentum in the direction of the fifth dimension *only if* its internal state changes, while in the present framework a massive particle has a momentum in the direction of the fifth dimension *even though* its internal state is constant.

Without further discussion we mention that Barrow has also published a 5D theory [28], but in that study a 5D mathematical space has been used to describe a 4D spacetime with compact 'large' dimensions—so from the *physical* point of view, Barrow's theory is not 5D.

4.3 5D kinematics of some physical processes

The objective of this section is to describe three kinds of processes—inertial motion of massive particles, Bremsstrahlung, and laser cooling—in the language of $\mathscr{C}_{SR}$. The statements are purely descriptive: there is no 'why' to the inertial motion or to the Bremsstrahlung.

4.3.1 Inertial motion of massive particles

Definition 4.20 (Inertial motion in $\mathscr{C}_{SR}$) For integers $n \in \mathbb{Z}$ and $k \in S_\omega$, in the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution is a process of **inertial motion** if and only if

(i) no γ-ray is absorbed at the discrete transition ${}^{EP}f_k^n \to {}^{NW}f_k^n$ at the position $X_{n,k}$: we thus have $p^\alpha_{n,k} = (\vec{p}^{(5\downarrow)}_{X_{n,k}})^\alpha$ as in Eq. (72) with $\Delta p^\alpha_{p,m} = 0$;

(ii) no γ-ray is emitted upon the discrete transition ${}^{NW}f_k^n \to {}^{NP}f_k^{n+1}$ at the position $X_{n+1,k}$: we thus have $\neg\mathbb{E}\gamma_k^{n+1}$ and, from Eqs. (69) and (70), $(\vec{p}^{(5\downarrow)}_{X_{n+1,k}})^\alpha = p^\alpha_{n,k}$.

□

Translated into terms of particles and events, this means for an inertial observer $\mathcal{O}$ that if a particle exhibits inertial motion between the events $\mathcal{E}_1 \xrightarrow{\mathcal{O}} (t_1, x_1, y_1, z_1, n_1)$ and $\mathcal{E}_2 \xrightarrow{\mathcal{O}} (t_2, x_2, y_2, z_2, n_2)$, $t_2 > t_1$ on its 5D world line ℓ, then the 5-momentum of the particle is a constant, and there is no event $\mathcal{E}_3 \xrightarrow{\mathcal{O}} (t_3, x_3, y_3, z_3, n_3)$ on ℓ with $t_2 > t_3 > t_1$ where a massless particle is emitted or absorbed. See Fig. 4 for an illustration with a spacetime diagram.

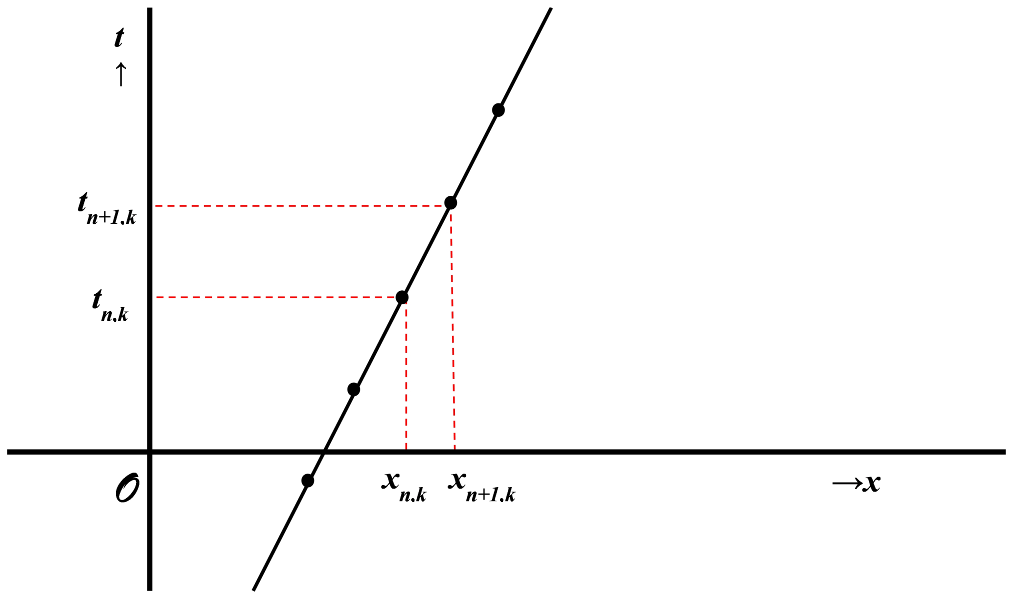

Figure 4: Spacetime diagram of a sequence of processes of inertial motion. Horizontally the spatial coordinates x of the 5D IRF of an inertial observer $\mathcal{O}$, vertically the time coordinates t. The five dots represent subsequent point-particle $s_k^n = {}^{EP}f_k^n$, the line segments connected by the dots represent subsequent time-like strings ${}^{NW}f_k^n$. Together this represents the k^{th} massive particle on its 5D world line ℓ_k; the constant slope of ℓ_k reflects the constant 5-momentum.

4.3.2 Bremsstrahlung

Definition 4.21 (Bremsstrahlung in $\mathscr{C}_{SR}$) For integers $n \in \mathbb{Z}$ and $k \in S_\omega$, in the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution is a process with **Bremsstrahlung** if and only if

(i) no γ-ray is absorbed at the discrete transition ${}^{EP}f_k^n \to {}^{NW}f_k^n$ at the position $X_{n,k}$: we thus have $p^\alpha_{n,k} = (\vec{p}^{(5\downarrow)}_{X_{n,k}})^\alpha$ as in Eq. (72) with $\Delta p^\alpha_{p,m} = 0$;

(ii) a γ-ray is emitted at the discrete transition ${}^{NP}f_k^{n+1} \to \gamma_k^{n+1}$ at the position $X_{n+1,k}$: we thus have $\mathbb{E}\gamma_k^{n+1}$ and, from Eqs. (69) and (70), $(\vec{p}^{(5\downarrow)}_{X_{n+1,k}})^\alpha := p^\alpha_{n,k} - \Delta p^\alpha_{n+1,k}$.

□

So as a simple example, consider that the point-particle ${}^{EP}f_k^n$ has 5-momentum $(E, p_x, 0, 0, m)$, such that $p_x > 0$ and $E^2 = (p_x)^2 + m^2$. At its transition to the time-like string ${}^{NW}f_k^n$, this 5-momentum is conserved, so at any point on the line segment occupied by the time-like string ${}^{NW}f_k^n$, the 5-momentum is also $(E, p_x, 0, 0, m)$. Upon the transition of the time-like string ${}^{NW}f_k^n$ to the new point-particle ${}^{NP}f_k^{n+1}$, the latter emits a γ-ray with 5-momentum $(\Delta E, \Delta p_x, 0, 0, 0)$ with $p_x > \Delta p_x > 0$ and $\Delta E = \Delta p_x$. Upon emission, the point-particle ${}^{NP}f_k^{n+1}$ then transforms into the new point-particle ${}^{EP}f_k^{n+1}$: its 5-momentum is then $(E', p_x - \Delta p_x, 0, 0, m)$ with $E' = (p_x - \Delta p_x)^2 + m^2 < E$.

Translated into terms of particles and events, this means for an inertial observer $\mathcal{O}$ that if a particle emits Bremsstrahlung between the events $\mathcal{E}_1 \xrightarrow{\mathcal{O}} (t_1, x_1, y_1, z_1, n_1)$ and $\mathcal{E}_2 \xrightarrow{\mathcal{O}} (t_2, x_2, y_2, z_2, n_2)$ on its 5D world line ℓ, $t_2 > t_1$, then the energy and spatial momentum of the particle decrease stepwise through the emission of massless particles (photons). See Fig. 5 for an illustration with a spacetime diagram.

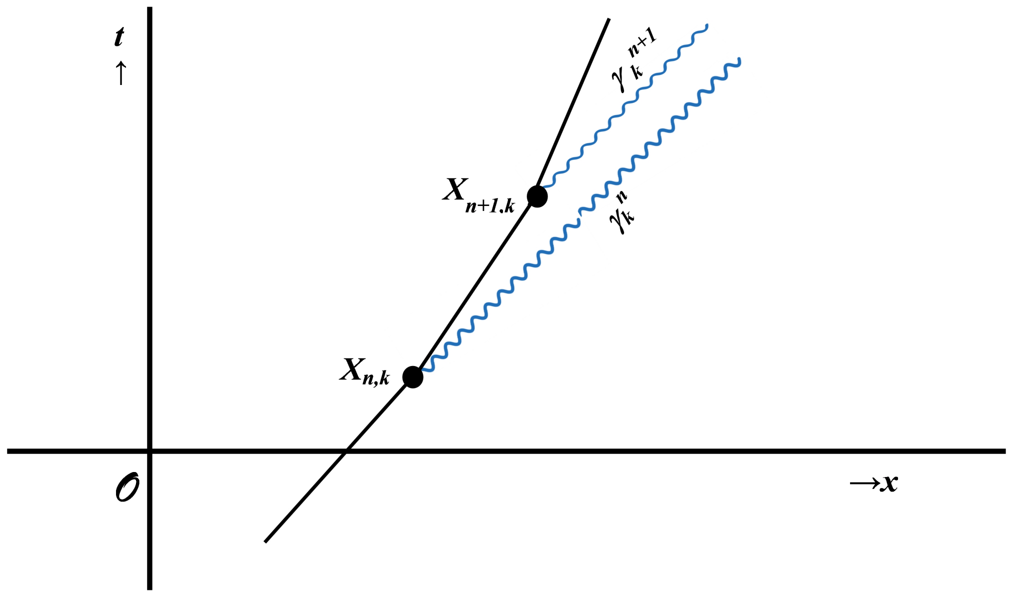

Figure 5: Spacetime diagram of subsequent processes with Bremsstrahlung. Horizontally the spatial coordinates x of the 5D IRF of an inertial observer $\mathcal{O}$, vertically the time coordinates t. The two dots represent subsequent point-particles $s_k^n = {}^{EP}f_k^n$ and $s_k^{n+1} = {}^{EP}f_k^{n+1}$, the line segments connected by the dots represent subsequent time-like strings ${}^{NW}f_k^{n-1}$, ${}^{NW}f_k^n$, and ${}^{NW}f_k^{n+1}$. The wavy blue lines represent emitted γ-rays γ_k^n and γ_k^{n+1}. Together this represents the k^{th} massive particle on its 5D world line ℓ_k, plus two emitted photons; the increasing slope of ℓ_k reflects the stepwise deceleration.

4.3.3 Laser cooling

Definition 4.22 (Laser cooling in $\mathscr{C}_{SR}$) For integers $n \in \mathbb{Z}$ and $k \in S_\omega$, in the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution is a process with **laser cooling** if and only if

(i) a γ-ray γ_m^p from a laser source is absorbed at the discrete transition ${}^{EP}f_k^n \to {}^{NW}f_k^n$ at the position $X_{n,k}$: for some $p \in \mathbb{Z}$ and $m \in S_\omega$ we thus have $p_{n,k}^\alpha = (\vec{p}_{X_{n,k}}^{(5\downarrow)})^\alpha + \Delta p_{p,m}^\alpha$ as in Eq. (72), but in particular with $E_{n,k}^{NW} < E_{n,k}^{EP}$ (decreasing energy);

(ii) no γ-ray is emitted upon the discrete transition ${}^{NW}f_k^n \to {}^{NP}f_k^{n+1}$ at the position $X_{n+1,k}$: we thus have $\neg\mathbb{E}\gamma_k^{n+1}$ and, from Eqs. (69) and (70), $(\vec{p}_{X_{n+1,k}}^{(5\downarrow)})^\alpha = p_{n,k}^\alpha$.

□

So as a simple example, consider that the point-particle ${}^{EP}f_k^n$ has 5-momentum $(E, p_x, 0, 0, m)$, such that $p_x > 0$ and $E^2 = (p_x)^2 + m^2$. At its transition to the time-like string ${}^{NW}f_k^n$, a γ-ray is absorbed with 5-momentum $(\Delta E, -\Delta p_x, 0, 0, 0)$ with $-\Delta p_x < 0$ and $\Delta E = \Delta p_x$. Then at any point on the line segment occupied by the time-like string ${}^{NW}f_k^n$, the 5-momentum is $(E', p_x - \Delta p_x, 0, 0, m)$ with $E' = (p_x - \Delta p_x)^2 + m^2 < E$.

Translated into terms of particles and events, this means for an inertial observer $\mathcal{O}$ that if a particle is laser cooled between the events $\mathcal{E}_1 \xrightarrow{\mathcal{O}} (t_1, x_1, y_1, z_1, n_1)$ and $\mathcal{E}_2 \xrightarrow{\mathcal{O}} (t_2, x_2, y_2, z_2, n_2)$ on its 5D world line ℓ, $t_2 > t_1$, then the energy and spatial momentum of the particle decrease stepwise through the absorption of massless particles (photons) emitted by a laser tube. See Fig. 6 for an illustration with a spacetime diagram.

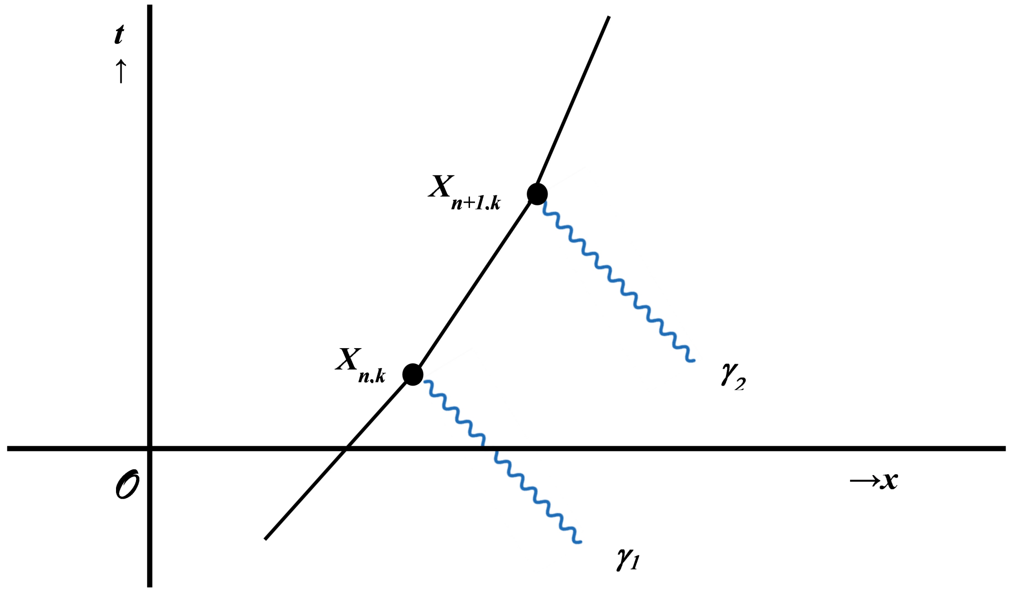

Figure 6: Spacetime diagram of subsequent processes with laser cooling. Horizontally the spatial coordinates x of the 5D IRF of an inertial observer $\mathcal{O}$, vertically the time coordinates t. The two dots represent subsequent point-particles $s_k^n = {}^{EP}f_k^n$ and $s_k^{n+1} = {}^{EP}f_k^{n+1}$, the line segments connected by the dots represent subsequent time-like strings ${}^{NW}f_k^{n-1}$, ${}^{NW}f_k^n$, and ${}^{NW}f_k^{n+1}$. The wavy blue lines represent γ-rays γ_1 and γ_2 from a laser source that are absorbed at the points $X_{n,k}$ and $X_{n+1,k}$, respectively. Together this represents the k^{th} massive particle on its 5D world line ℓ_k, plus two absorbed photons; the increasing slope of ℓ_k reflects the stepwise deceleration by laser cooling.

4.4 Intended relevance of the EPT for physics

The objective of this section is to state the intended relevance of the EPT for physics. Suppose that both issues mentioned in the Introduction are solved, that is, suppose that repulsive gravity has been detected and it has been proved (somehow) that the EPT agrees with existing knowledge; then we know *that* the EPT is relevant for physics, but it still remains to be stated *how* it is relevant.

This is not trivial, because on the one hand the EPT, as a mathematically abstract theory, makes only predictions that are physically abstract too, while on the other hand physics is a branch of exact science that is primarily interested in exact, i.e. mathematically concrete, predictions. One of the consequences thereof is that any *correctness* of EPT cannot be gauged by verifying its predictions: the accepted definition of correctness of a theory in physics is that a theory is correct if and only if all of its predictions are true [29], but due to the abstract nature of the predictions of the EPT its correctness is then impossible to gauge experimentally. So from a pragmatic point of view, this notion of correctness does not apply to the EPT. However, based on the soundness theorem of first-order predicate logic, we can define a notion of *physical soundness* for the EPT, which we can apply instead of correctness. This requires the following definition of a 'null category':

Definition 4.23 A **null category** is a category of which all objects are null objects, that is, are both initial and terminal objects. □

A corollary of Def. 4.23 is thus that any full subcategory of the present categorical model $\mathscr{C}_{SR}$, whose objects are a model class $[M_{\mathbb{Z},\omega,\mathcal{O}}]_\sim$ and whose arrows are the corresponding isomorphisms, is a null category. Proceeding, based on the soundness theorem for first-order logic, to wit: $(\psi_1, \ldots, \psi_n \vdash \chi) \Rightarrow (\psi_1, \ldots, \psi_n \models \chi)$, we then define 'physical soundness' of the EPT as follows:

Definition 4.24 The EPT is **physically sound** if and only if it has a categorical model $\mathscr{C}$ such that $\mathscr{C}$ is a null category whose objects are correct models of the world of an observer. (The objects of $\mathscr{C}$ are then a model class.) □

Here 'correct' is meant in the sense of the EPR-paper. With this definition the physical soundness of the EPT can be gauged by verifying concrete predictions of models of the EPT. So *to believe in the EPT* is *to believe that the EPT is physically sound* in the above sense.

That being said, the EPT is thus supposed to be physically sound, and the intended relevance of the EPT for physics is then that the EPT is intended as a Grand Unifying Scheme. This can be defined more precisely as follows:

Definition 4.25 (Grand Unifying Scheme) The EPT is a **Grand Unifying Scheme** if and only if it has a categorical model $\mathscr{C}$ such that <u>all</u> observations in the realm of physics can be formulated as predictions in the language of that category $\mathscr{C}$. □

This idea of a Grand Unifying Scheme is thus related to Van Fraassen's idea of *empirical adequacy*, introduced in [30]: the EPT is a Grand Unifying Scheme if and only if it has a model class that is empirically adequate—a model class is thus empirically adequate if and only if all observations in the realm of physics can be described as predictions in the language of that model class.

This idea of a Grand Unifying Scheme should, thus, **absolutely not** be confused with the idea of a Grand Unified Theory: a Grand Unified Theory is a merger of the three gauge interactions of the Standard Model (electromagnetic, weak, strong) in a single interaction model. So, a Grand Unified Theory is thus confined to the framework of the Standard Model, while the above definition of a Grand Unifying Scheme does not assume that objects of the category (which are models of the EPT) have to be formalized in the framework of quantum field theory.

That being said, the present result—the categorical model $\mathscr{C}_{SR}$—does not prove that the EPT indeed is a Grand Unifying Scheme. The intention is therefore to develop successors $\mathscr{C}_1, \mathscr{C}_2, \mathscr{C}_3, \ldots$ of $\mathscr{C}_{SR}$ that are theoretically and empirically progressive—here a categorical model $\mathscr{C}_{n+1}$ is *theoretically progressive* compared to a categorical model $\mathscr{C}_n$ when not only <u>all</u> observations, which could be expressed as predictions in the language of $\mathscr{C}_n$, can also be expressed in the language of $\mathscr{C}_{n+1}$ but also <u>some</u> observations, which could not be expressed as predictions in the language of $\mathscr{C}_n$, can be expressed in the language of $\mathscr{C}_{n+1}$; and a categorical model $\mathscr{C}_{n+1}$ is *empirically progressive* compared to a categorical model $\mathscr{C}_n$ when in the framework of $\mathscr{C}_{n+1}$ predictions can be formulated that are impossible in the framework of $\mathscr{C}_n$ <u>and</u> some of these predictions have been verified. The idea is then that this sequence of successors $\mathscr{C}_1, \mathscr{C}_2, \mathscr{C}_3, \ldots$ of $\mathscr{C}_{SR}$ converges to a categorical model $\mathscr{C}_\infty$ that satisfies Def. 4.25. This is a fundamentally new research program in theoretical physics, where the term 'research program' is used in the sense meant by Lakatos, cf. [31].

4.5 Conclusions

The main conclusion is that this study has proven that the EPT agrees with SR by proving that the EPT has a 5D categorical model $\mathscr{C}_{SR}$ that incorporates SR—this is a new mathematical-logical technique in physics. This result renders the EPT consistent with the outcome of real-world experiments and observations that can be described as predictions of SR—examples are the null result of the Michelson-Morley experiment [32], and the observed prolonged lifetime of fast muons [33]. In addition, it has been shown that laser cooling and Bremsstrahlung can be described in the language of the categorical model $\mathscr{C}_{SR}$.

The main advancement is that we henceforth *know* that the EPT agrees with the knowledge of the physical world obtained from the experimentally confirmed predictions of SR: before this study this was unknown, and a negative result could have been that a categorical model of the EPT incorporating SR is nonexisting—which would have implied that the EPT does not agree with this knowledge of the physical world. From the perspective of the semantic view on theories, the objects of the categorical model $\mathscr{C}_{SR}$ correspond to a 5D account of SR: although aspects hereof can be found in existing 5D accounts of relativity (not necessarily limited to SR), we conclude—in particular because of the discrete nature of the individual processes—that this is a new 5D account of SR. Nevertheless, the present study doesn't purport to yield an advancement in relativity theory; for example, the present 5D SR does not shed a new light on any of the paradoxes of SR.

A limitation of this study is that it has been focused purely at demonstrating the agreement of the EPT with SR, and with SR alone. A question that is therefore still left unanswered is whether or not the EPT agrees with the knowledge of the physical world obtained from the experimentally confirmed predictions of modern, relativistic interaction theories. Further research is therefore to be aimed at establishing whether or not the categorical model $\mathscr{C}_{SR}$ has a finite sequence of theoretically and empirically progressive successors $\mathscr{C}_1, \mathscr{C}_2, \mathscr{C}_3, \ldots$ that converges to a category $\mathscr{C}$, such that all observations in the realm of physics can be formulated as predictions in the language of that category $\mathscr{C}$. This is a fundamentally new research program in theoretical physics, aimed at exploring the potential of the EPT as a candidate for a Grand Unifying Scheme.

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